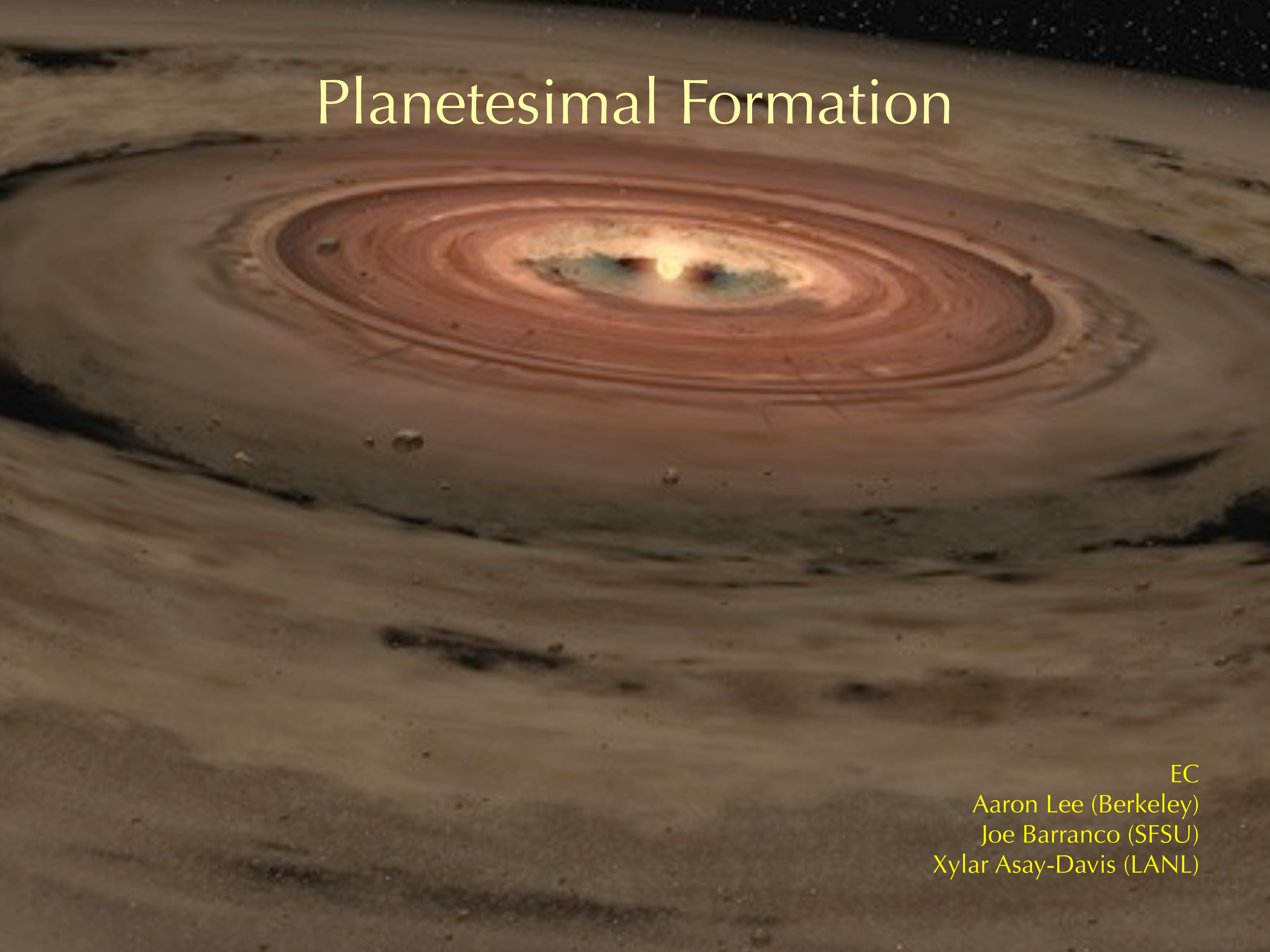

Planetesimal Formation

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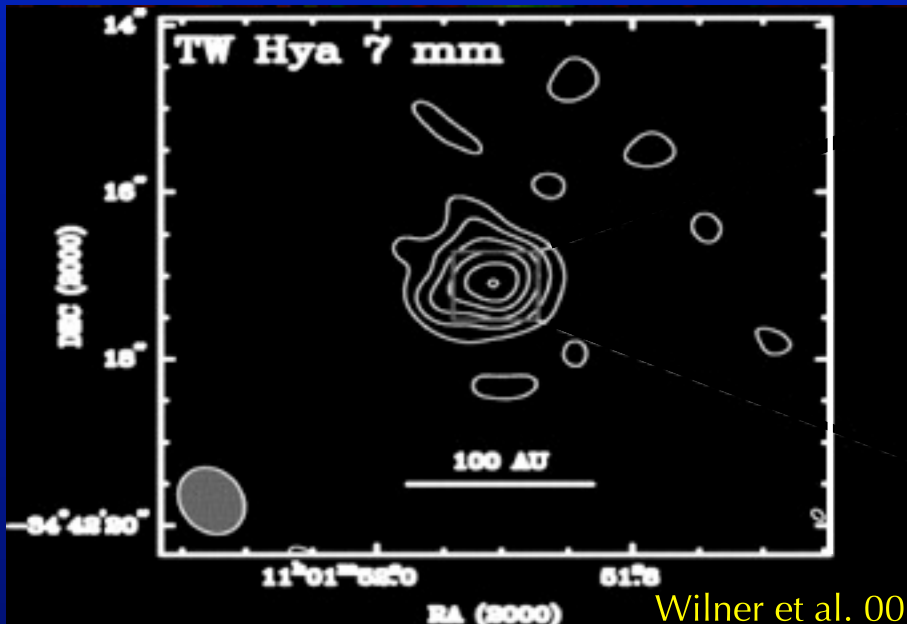
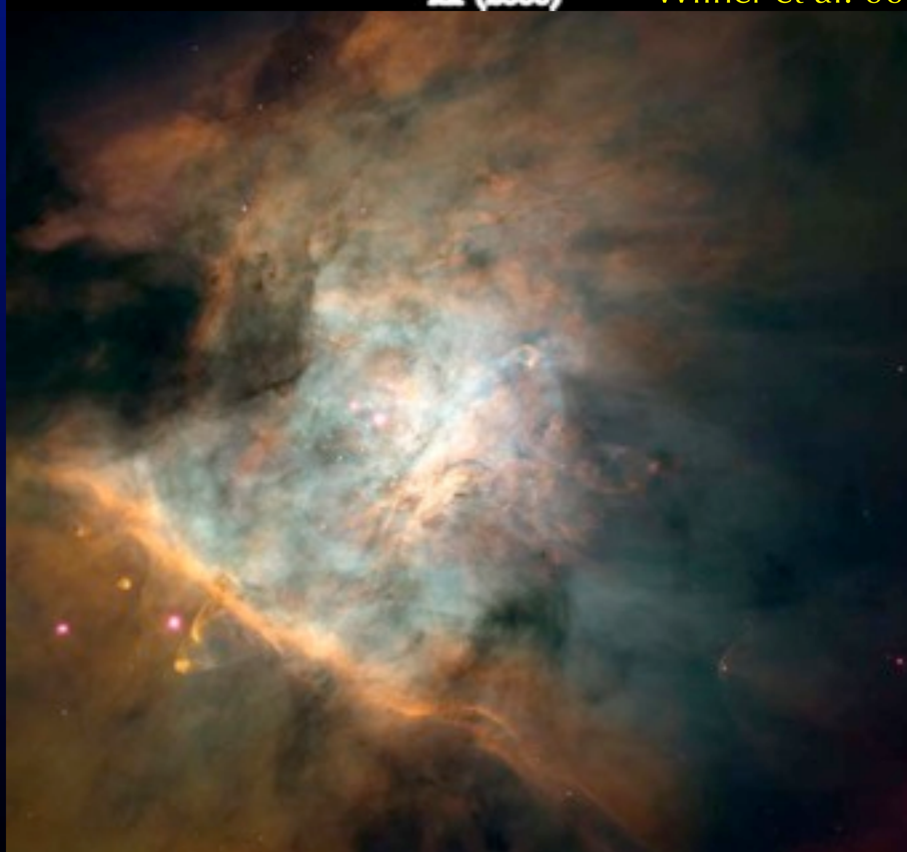
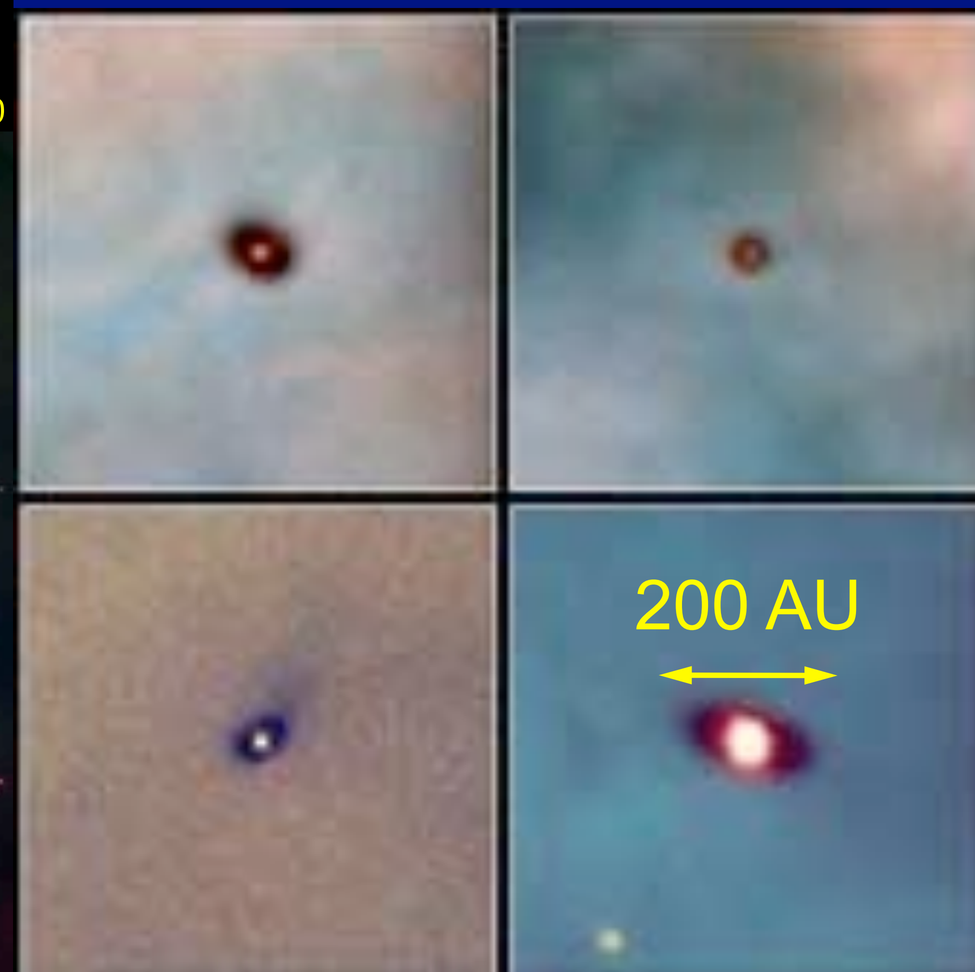
Protoplanetary Disks

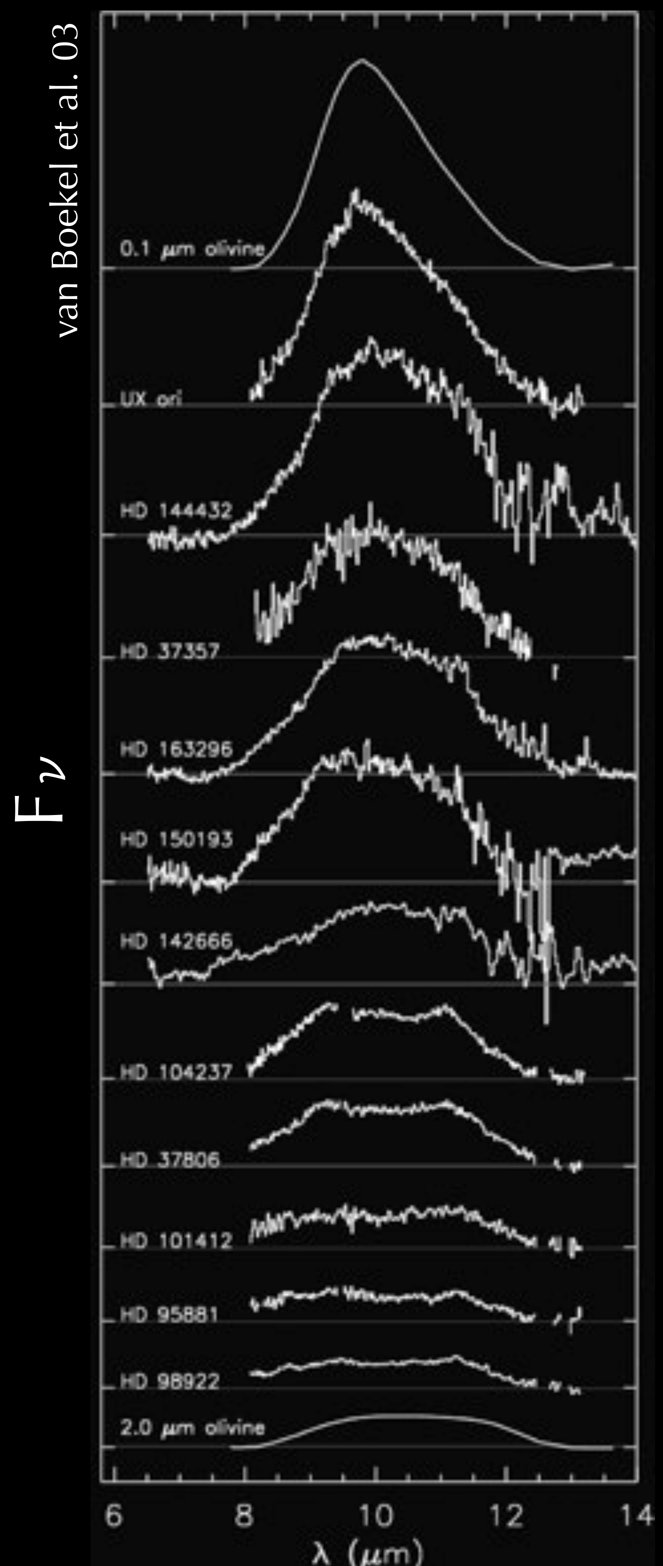
disk mass ~ 0.001-0.1 stellar mass



Wilner et al. 00

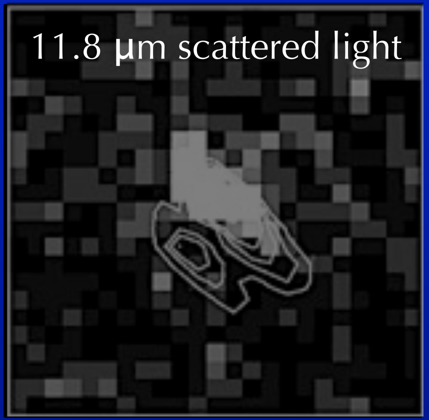






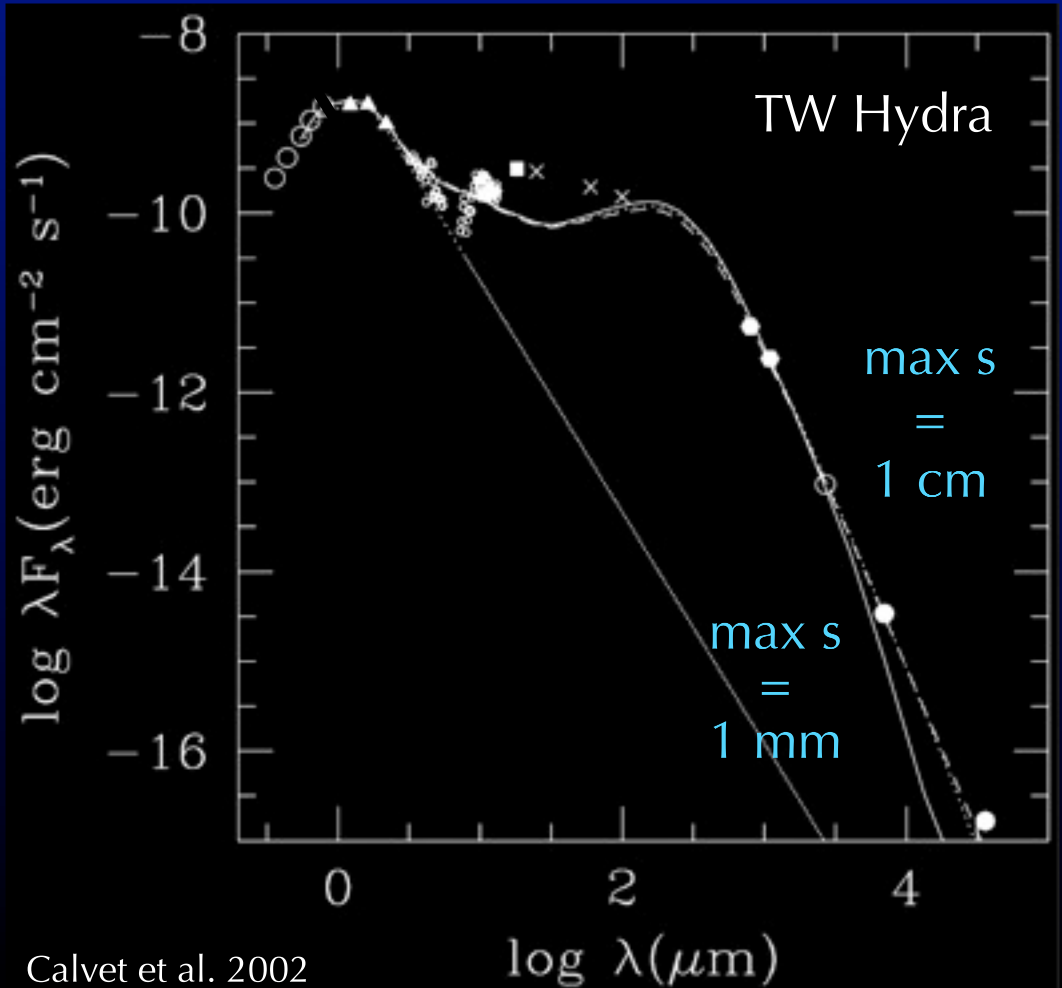
van Boekel et al. 03

Disk surfaces at ~10 AU: Growth to a few microns



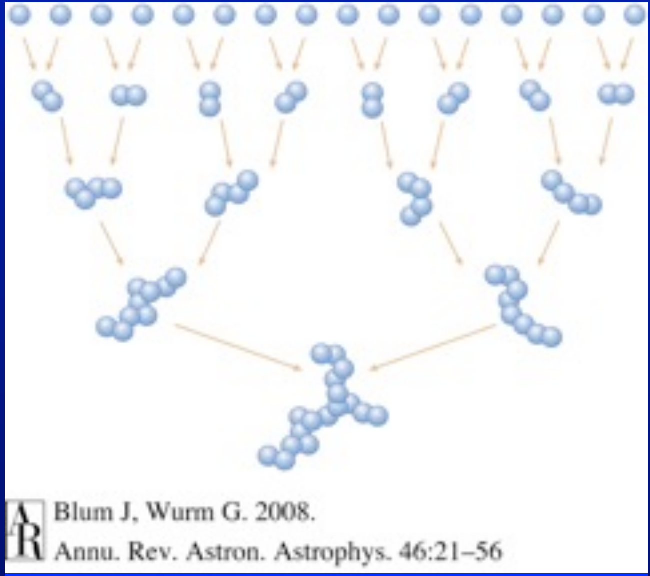
McCabe, Duchene, & Ghez 03

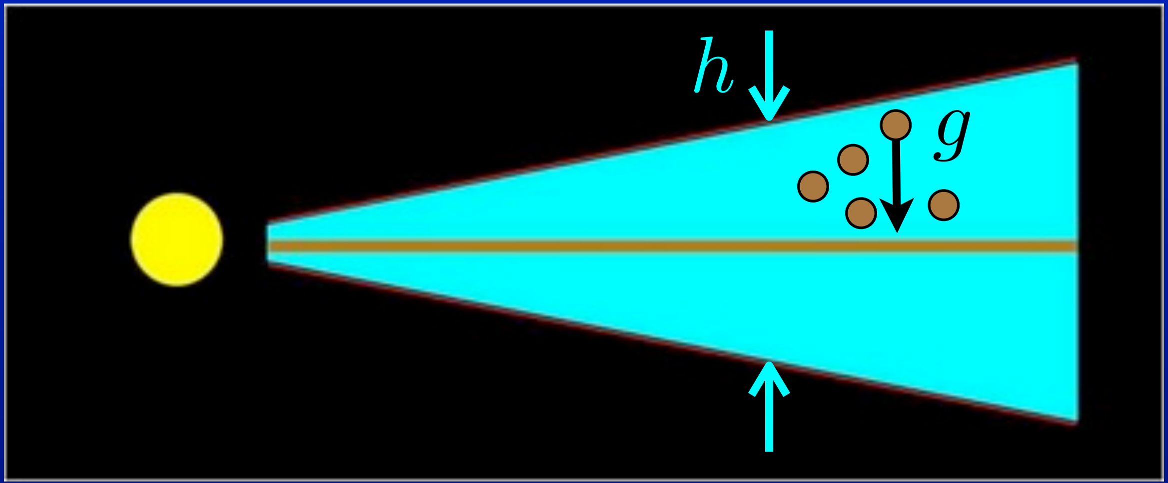
Disk interior at ~100 AU: Growth to a few cm

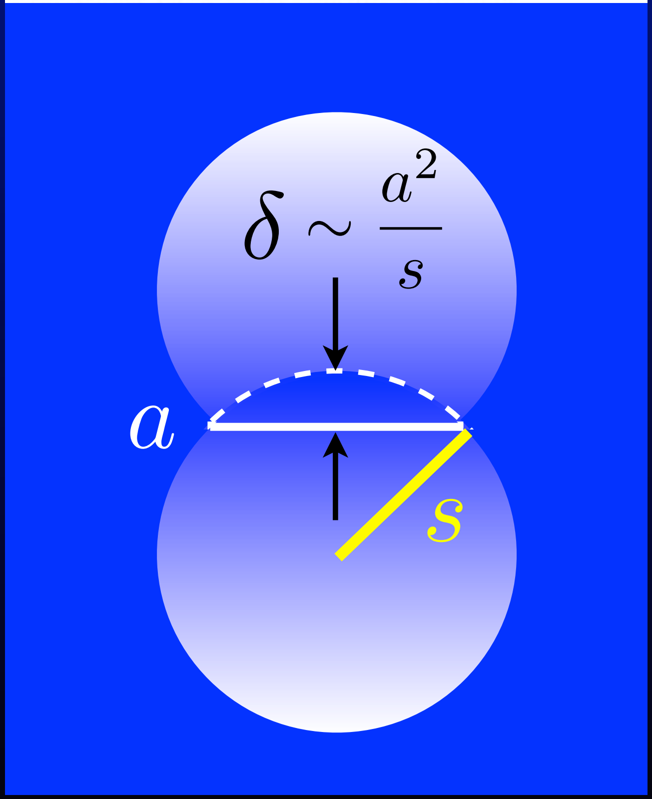


Calvet et al. 2002

Grain growth







Sticking $v_{\rm stick} \sim 1\,{\rm m/s}$ for $s \sim 1\,\mu{\rm m}$

Hertz + surface tension $\longrightarrow v_{\rm stick} \sim 4\frac{\gamma^{5/6}}{E^{1/3}\rho^{1/2}s^{5/6}}$

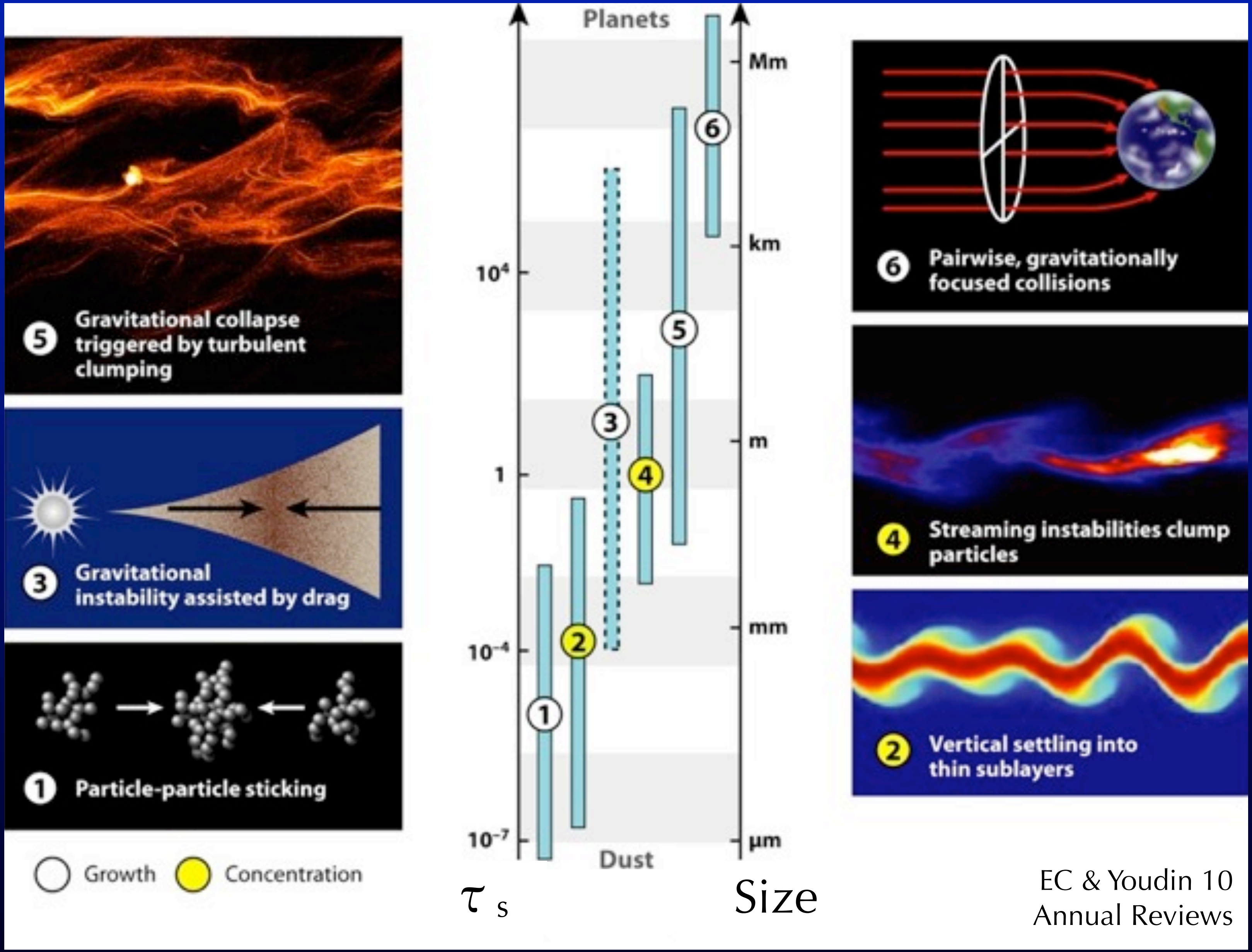
Terminal $v_{\rm term} \sim \frac{\rho}{\rho_g}\Omega s$

$\sim 1\,{\rm m/s}$ for $s \sim 10\,{\rm cm}$

Sticking up to, but not beyond, cm sizes

Chokshi et al. 93, Blum & Wurm 08

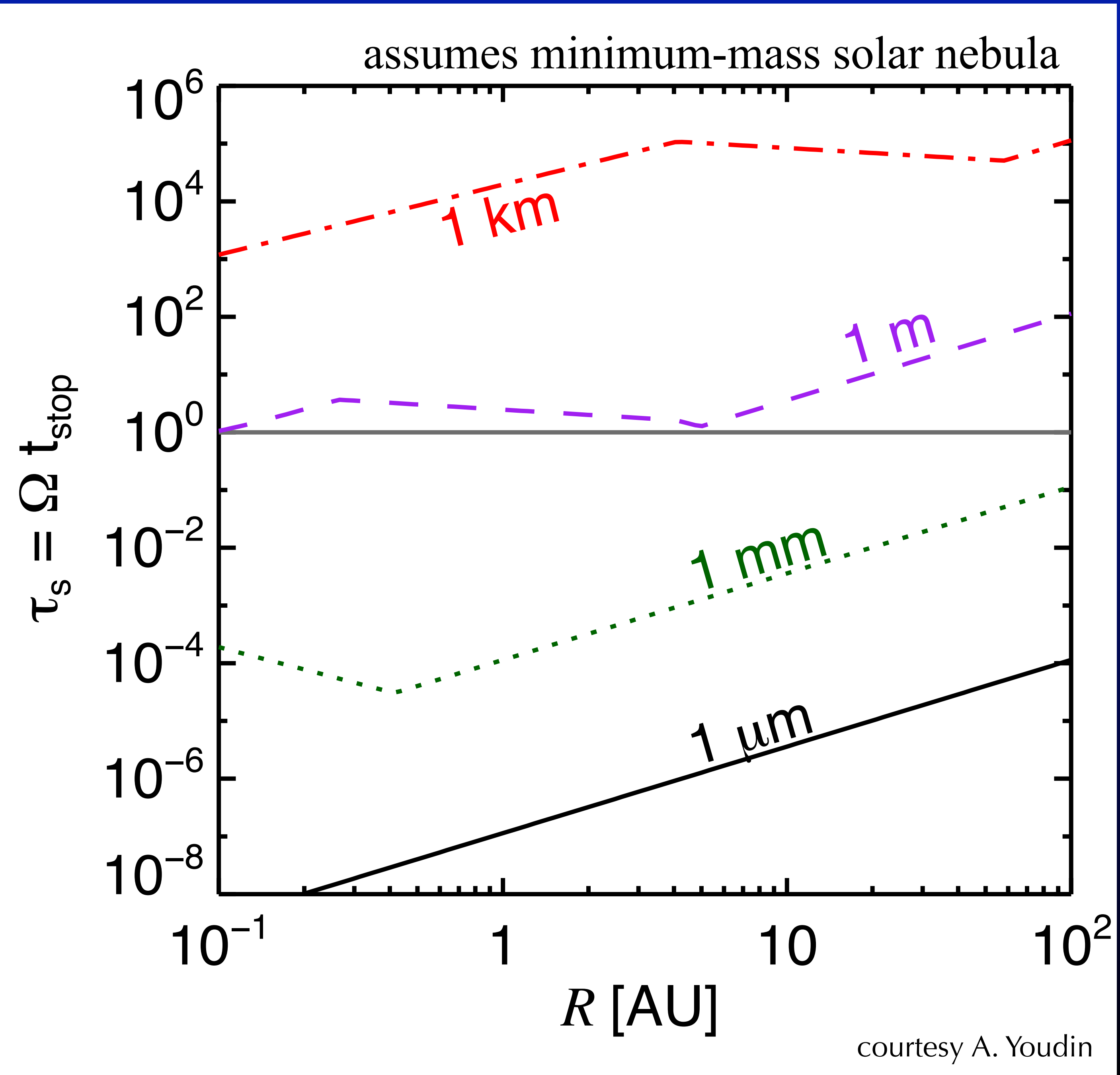
Climbing the size ladder



Grain stopping time $t_{stop} \equiv m v_{rel} / F_{drag}$

Dimensionless stopping time $\tau_s \equiv \Omega_{Kepler}\, t_{stop}$

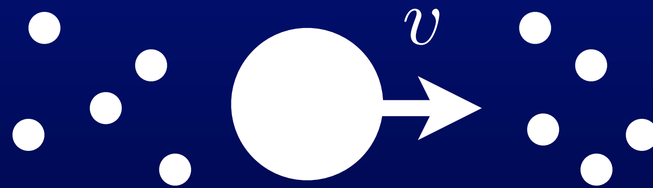
Gas-particle entrainment



e.g., Epstein drag

$$\lambda_{\rm mfp} > s$$

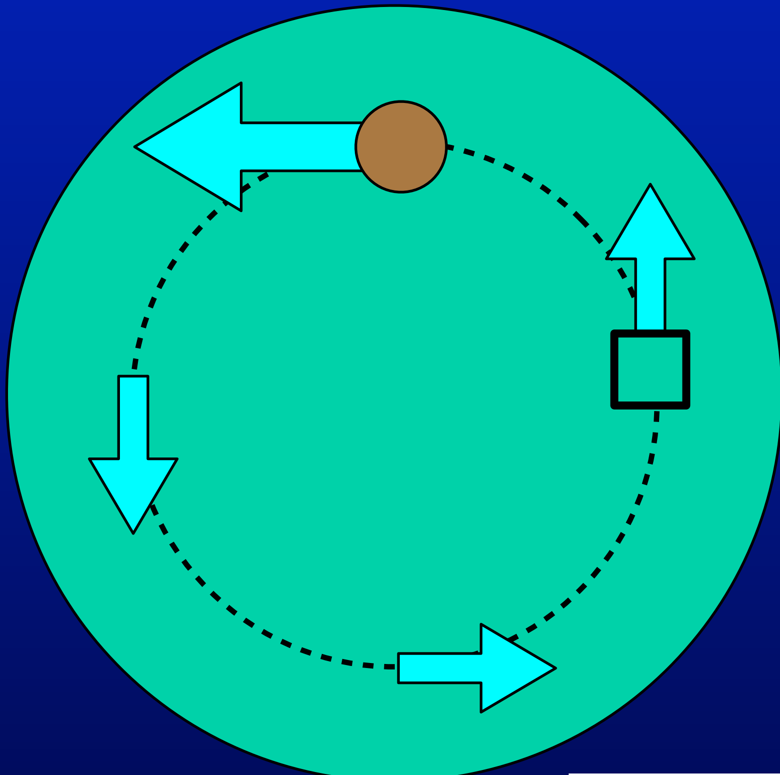
$$v < c_s$$



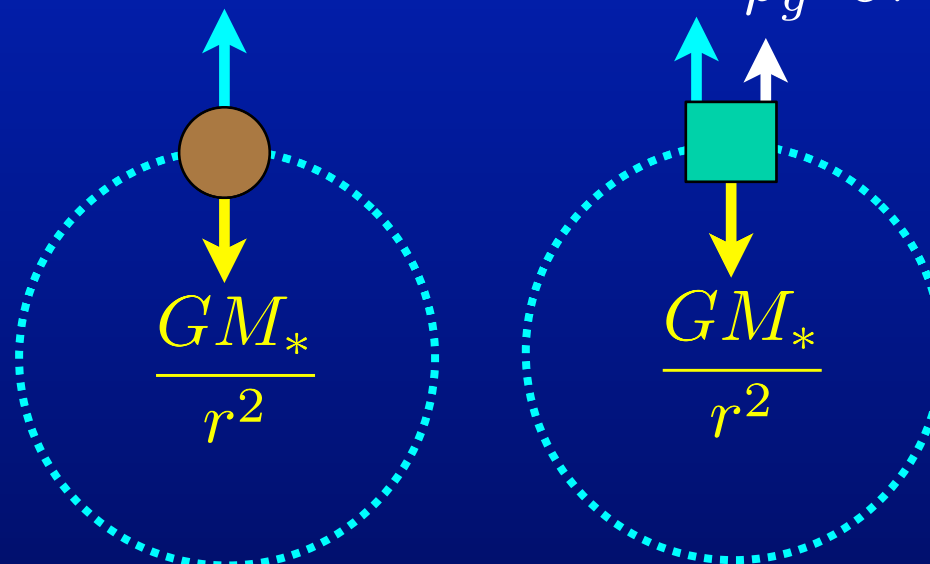
$\rho(c_s - v)^2$ $\rho(c_s + v)^2$

$$\Rightarrow F_{\rm drag} \sim \rho c_s v \times \pi s^2$$

Radial drift from headwind



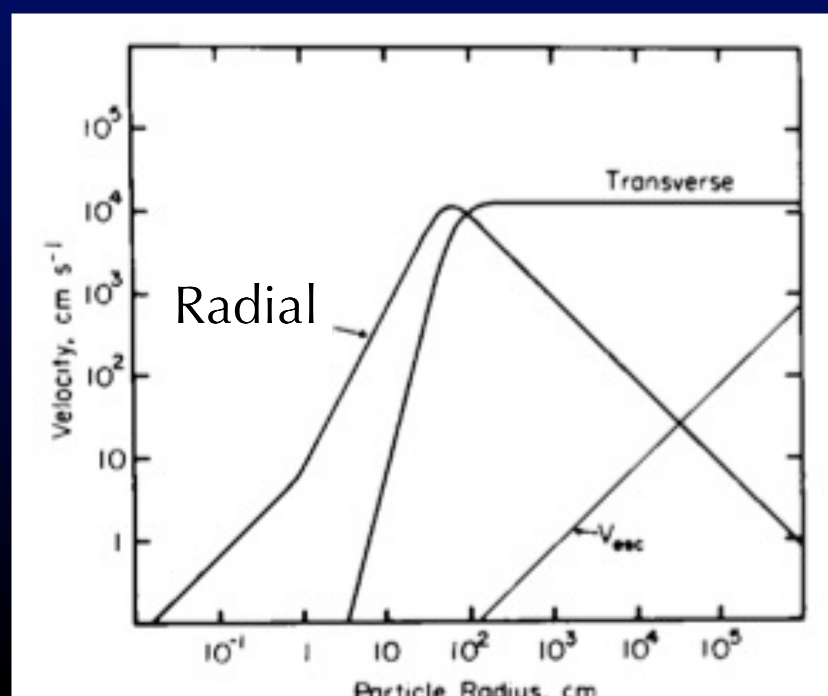
$$\Omega_K^2 r \qquad \Omega^2 r \quad \frac{-1}{\rho_g}\frac{\partial P}{\partial r}$$



$$\frac{GM_*}{r^2} \qquad \frac{GM_*}{r^2}$$

$$\therefore\ \Omega < \Omega_K$$

Meter-sized boulders drift inward from 1 AU within 100 yr



Weidenschilling 77
Nakagawa, Sekiya, & Hayashi 86

Gravitational instability

Disk annulus can fragment if Toomre Q ~ 1

$$\langle \rho \rangle > \rho_{\rm Toomre} \sim \frac{M_*}{2\pi r^3}$$

$$> 10^{-7}\,{\rm g\,cm^{-3}}$$

Clump can resist tidal shear if Roche unstable

$$\rho > \rho_{\rm Roche} \sim \frac{3.5 M_*}{r^3}$$

$$> 2 \times 10^{-6}\,{\rm g\,cm^{-3}}$$

$$\text{if } \Sigma_g \sim 2000\,{\rm g\,cm^{-2}}\ ({\rm minimum-mass\,solar\,nebula})$$

$$\text{if } \Sigma_d/\Sigma_g \sim 10^{-2}\ ({\rm height-integrated\,solar\,metallicity})$$

$$\text{then } \rho \sim \frac{\Sigma_g}{h_g} + \frac{\Sigma_d}{h_d}$$

$$\sim 3 \times 10^{-9}{\rm g\ cm^{-3}} \qquad \text{if } h_d \sim h_g$$

<u>Toomre unstable</u>

$$\rho_d/\rho_g \sim 30$$

<u>Roche unstable</u>

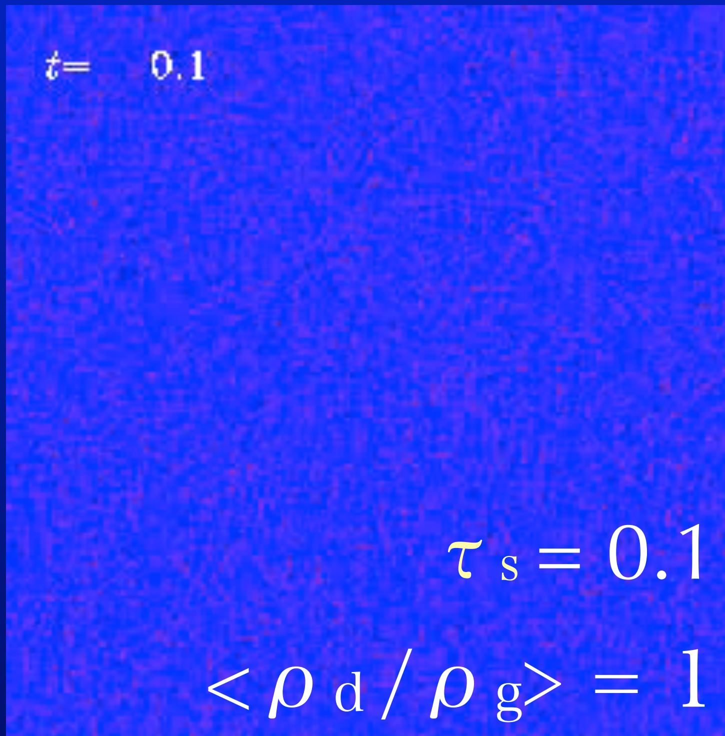
$$\rho_d/\rho_g \sim 600$$

"Streaming" instability

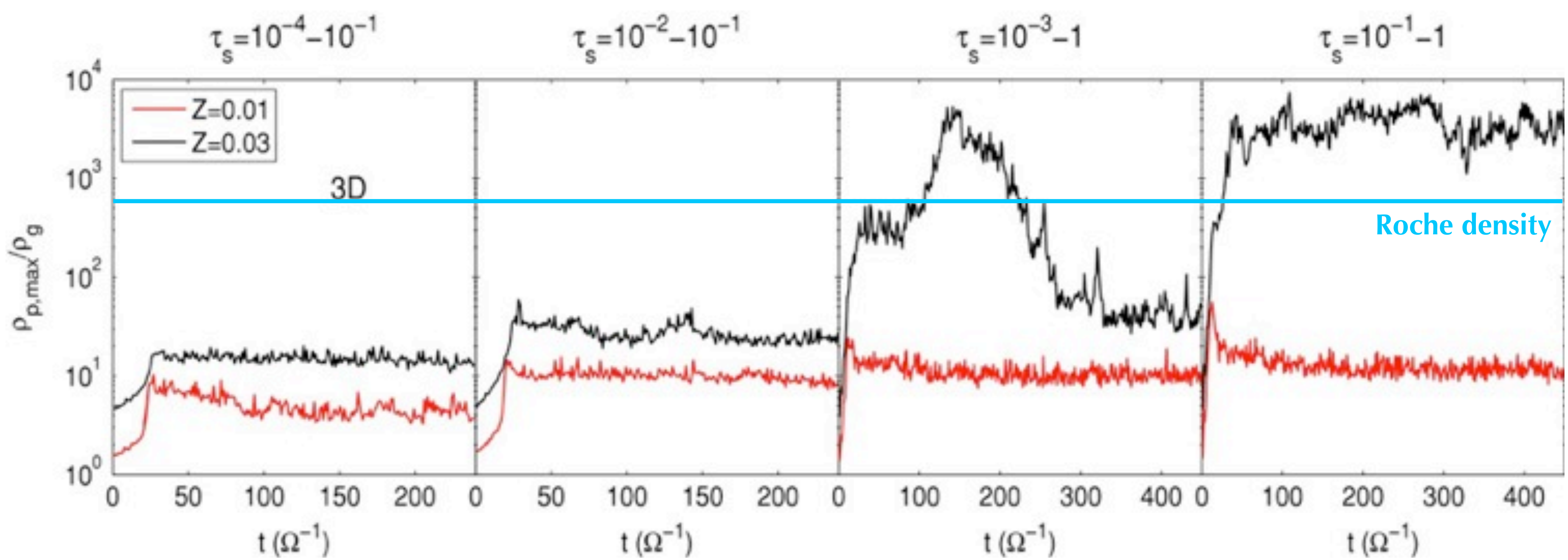
= linear instability between two fluids interacting frictionally in a disk

growth rates peak for $\tau_s \sim 1$

(marginally coupled bodies)



Youdin & Goodman 05; Johansen & Youdin 07



Bai & Stone 2011

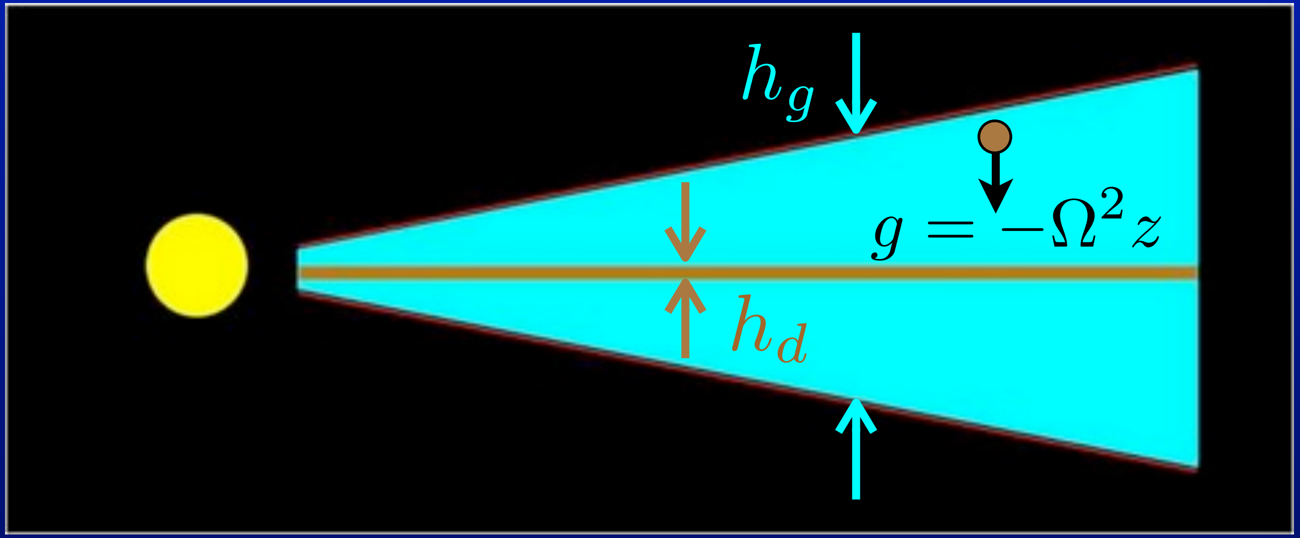
Gravitational instability in the $\tau_s << 1$ (small particle) limit

Self-gravity important when

$$\rho > \rho_{\mathrm{Toomre}} \sim \frac{M_*}{2\pi r^3}$$

$$> 10^{-7}\,\mathrm{g\,cm^{-3}}$$

$$\text{at } r = 1\,\mathrm{AU}$$



$$\text{if } \Sigma_g \sim 2000\,\mathrm{g\,cm^{-2}}\ (\text{minimum} - \text{mass solar nebula})$$

$$\text{if } \Sigma_d/\Sigma_g \sim 10^{-2}\ (\text{height} - \text{integrated solar metallicity})$$

$$\text{then } \ \rho \sim \frac{\Sigma_g}{h_g} + \frac{\Sigma_d}{h_d}$$

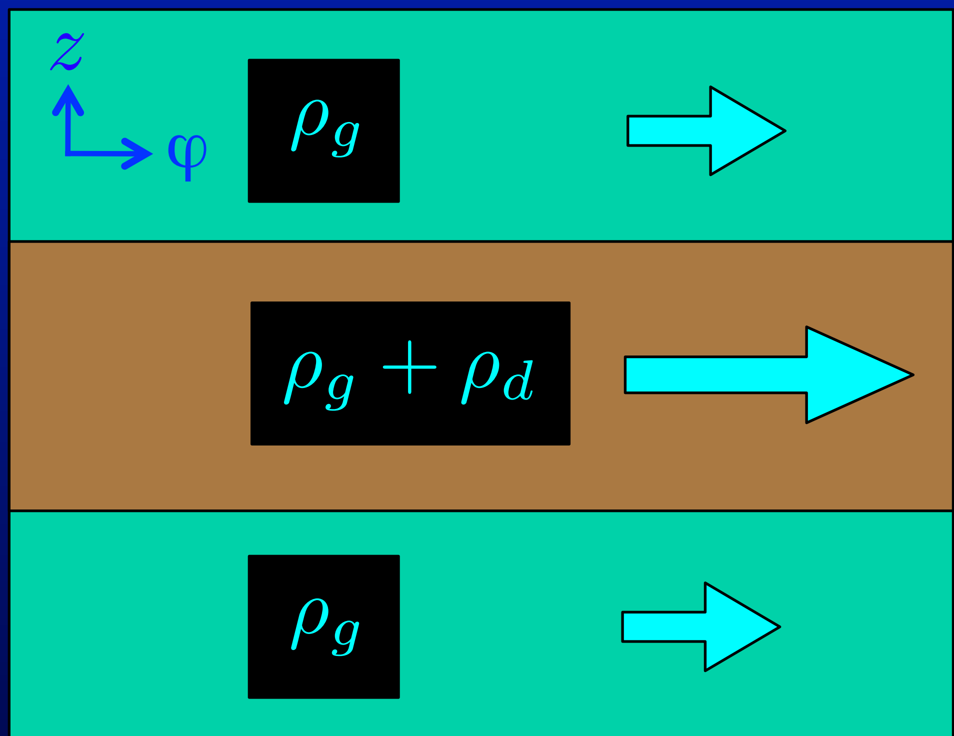
$$\sim 3 \times 10^{-9}\mathrm{g\ cm^{-3}} \qquad \text{if } \ h_d \sim h_g$$

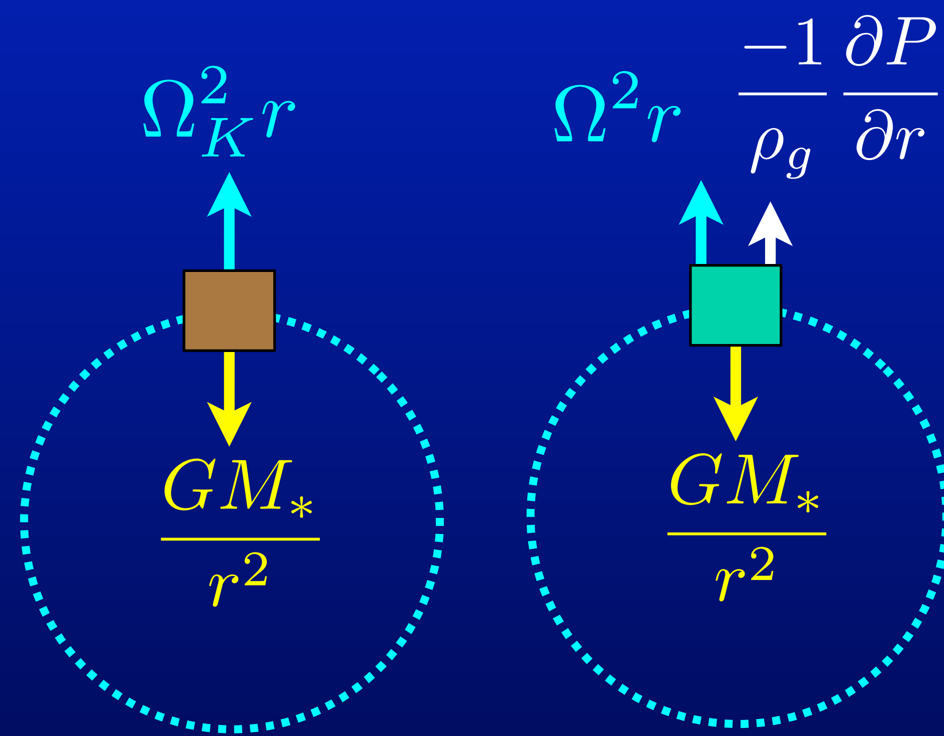
$$\sim 10^{-7}\,\mathrm{g\,cm^{-3}} \qquad \text{if } \ h_d \sim 5 \times 10^{-4} h_g$$

Can the settled "sublayer" achieve
Toomre density $< \rho_d / \rho_g > \sim 30$?

Safronov 1969
Goldreich & Ward 1973

Kelvin-Helmholtz instability may limit dust settling





$$\therefore\ \Omega < \Omega_K$$

$$\Delta v \sim c_s \frac{c_s}{v_\mathrm{K}} \sim 25\,\mathrm{m/s}\ \text{ nearly independent of } r$$

Weidenschilling 1980

Necessary criterion for K-H instability in Cartesian shear flow:

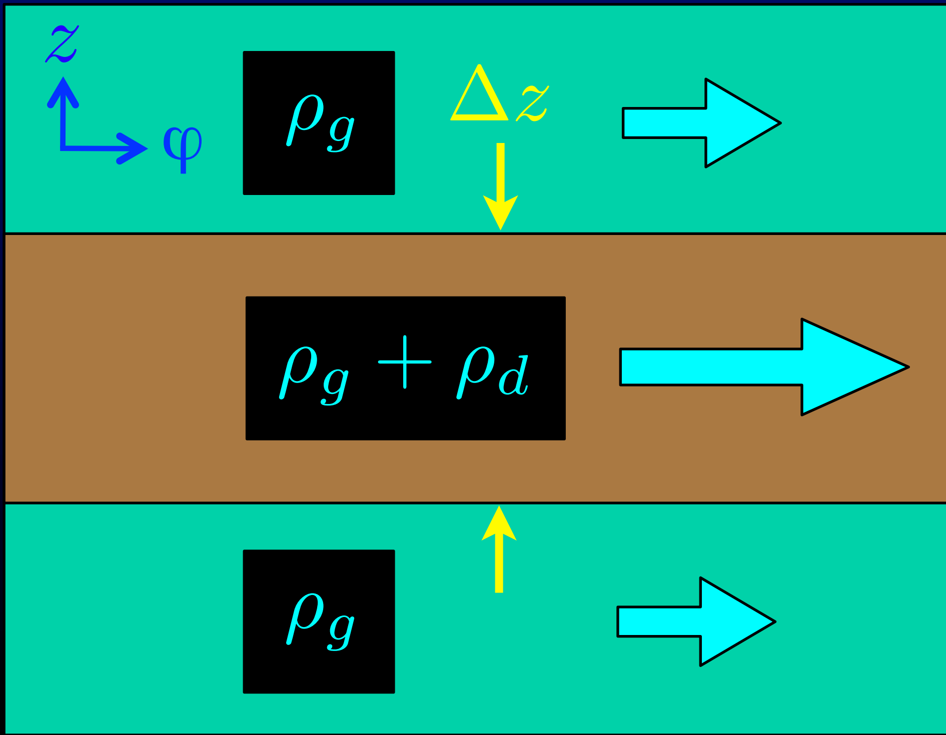
$$\text{Richardson} \quad Ri \equiv \frac{g\,\partial \ln \rho / \partial z}{(\partial v / \partial z)^2} < Ri_{\text{crit}} = \frac{1}{4}$$

$$= \frac{\omega^2_{Brunt}}{(\partial v / \partial z)^2}$$

If $Ri = 1/4$,

then $\Delta z \sim \dfrac{\Delta v}{\Omega} \sim 10^{-2} h_{\text{g}}$

$\rho_d / \rho_g \sim 1$



For small particles well coupled to gas ($\tau_s << 1$):

1. Is the Richardson criterion a good predictor of stability?

    - Doesn't formally apply because flow is 3D and rotational
    - Brunt vs. vertical shear vs. Coriolis vs. Kepler shear
    - Coriolis is destabilizing
    - Kepler shear is stabilizing

2. How does maximum dust density ρ_d depend on bulk (height-integrated) metallicity Σ_d/Σ_g?

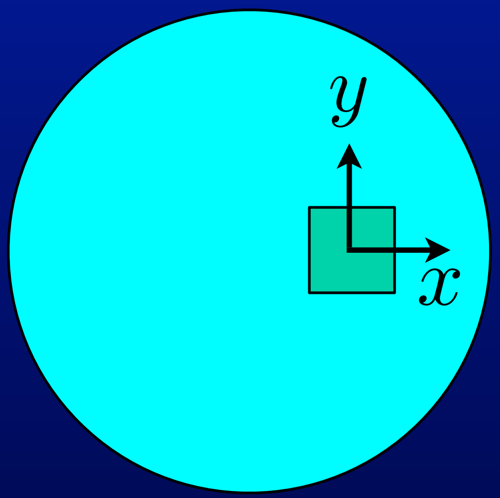
    - Disk metallicity may be supersolar
    - Host stars of extrasolar planets tend to be metal-rich
    - Planets themselves are metal-rich

Ishitsu & Sekiya 03; Gomez & Ostriker 05; Johnson et al. 10; Guillot et al. 06

Well-coupled gas and dust in a shearing box

$$\frac{\partial v_x}{\partial t} + \mathbf{v}\cdot\nabla v_x = \frac{-1}{\rho + \rho_d}\frac{\partial P}{\partial x} + 2\Omega_0 v_y + 2q\Omega_0^2 x \quad - \frac{\left(\frac{\partial P}{\partial x}\right)_0}{\rho + \rho_d}$$

$$\frac{\partial v_y}{\partial t} + \mathbf{v}\cdot\nabla v_y = \frac{-1}{\rho + \rho_d}\frac{\partial P}{\partial y} - 2\Omega_0 v_x$$



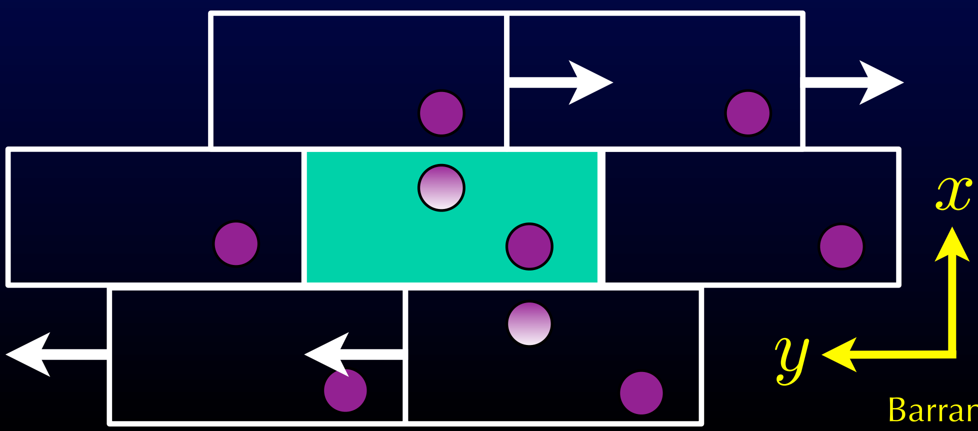
$$\frac{\partial v_z}{\partial t} + \mathbf{v}\cdot\nabla v_z = \frac{-1}{\rho + \rho_d}\frac{\partial P}{\partial z} - \Omega_0^2 z$$

anelastic

$$\cancel{\frac{\partial \rho}{\partial t}} + \nabla\cdot(\rho_0 \mathbf{v}) = 0 \qquad \frac{\partial \rho_d}{\partial t} + \nabla\cdot(\rho_d \mathbf{v}) = 0$$

$$\frac{\partial \varepsilon}{\partial t} + \mathbf{v}\cdot\nabla\varepsilon = -(P+\varepsilon)\nabla\cdot\mathbf{v}$$

$$P = (\gamma - 1)\varepsilon$$



Barranco 09

Code limitations (so far):

1. No self-gravity
   - Use Toomre density as guide for onset of gravitational instability
2. Dust and gas are perfectly coupled
   - Restricted to studying stability of given initial conditions

Numerical simulations of dense midplanes

Initial conditions:
Spatially constant Ri

$$\mu(z) \equiv \frac{\rho_d}{\rho_g}(z)$$

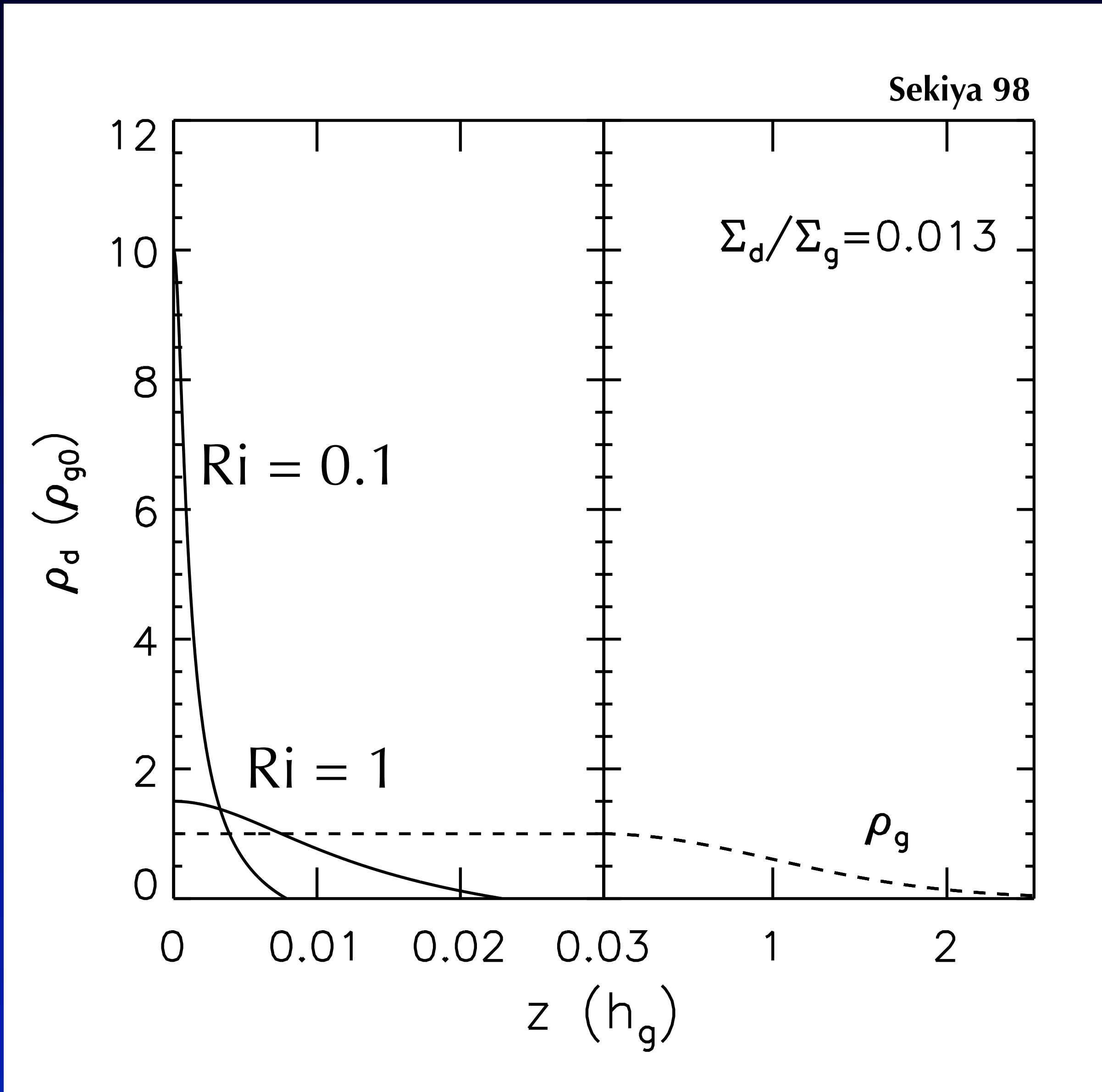
$$= \left[\frac{1}{1/(1+\mu_0)^2 + (z/z_d)^2}\right]^{1/2} - 1$$

where $z_d = \frac{Ri^{1/2}\Delta v}{\Omega}$

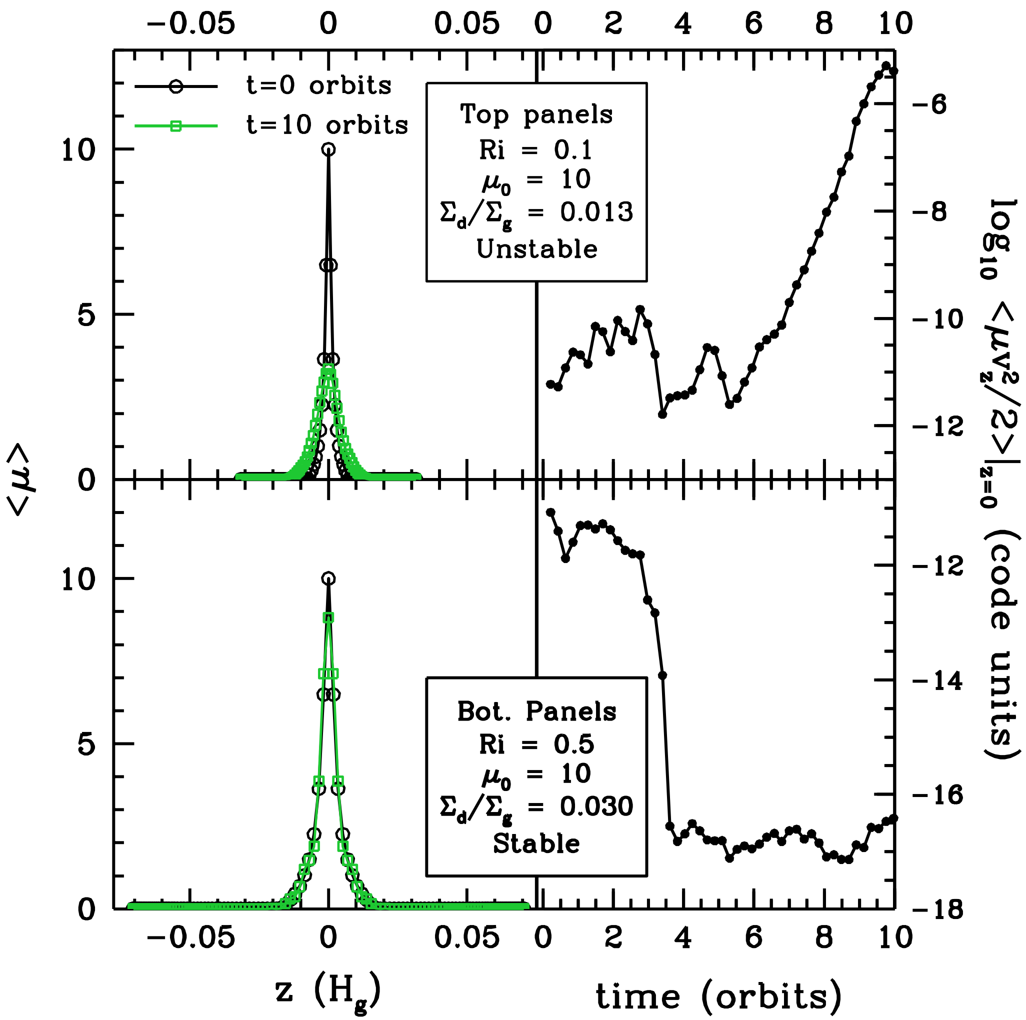
Input parameters:

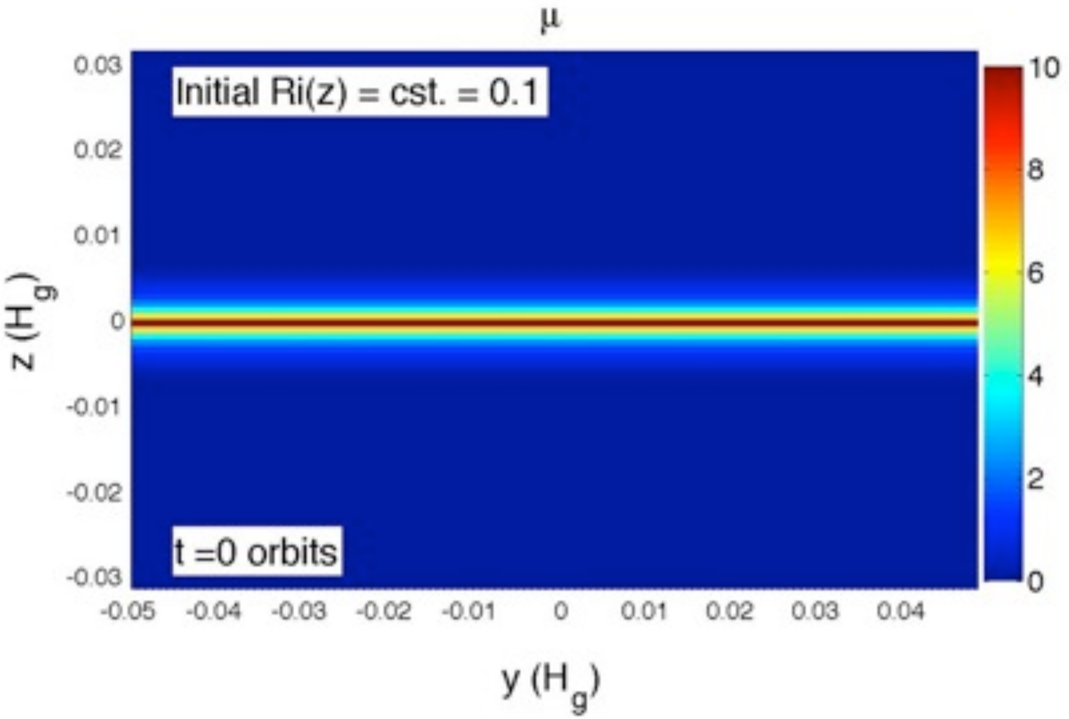
$(\mathrm{Ri},\ \mu_0)$

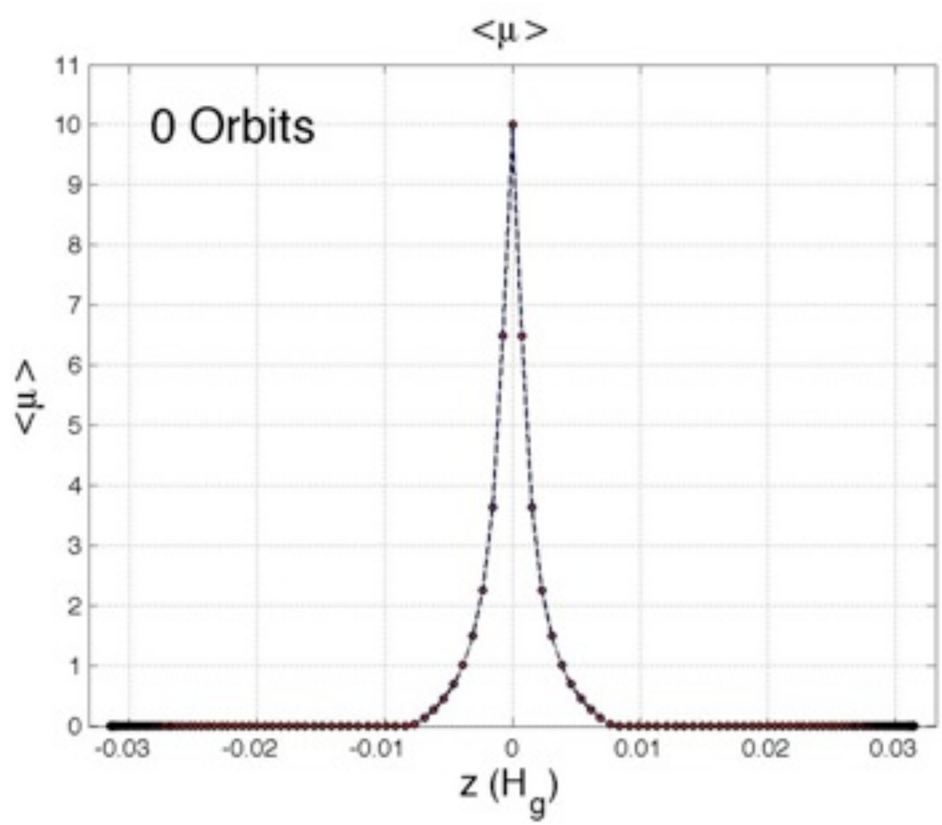
$(\mathrm{Ri},\ \Sigma_d/\Sigma_g) \longleftrightarrow (\mu_0,\ \Sigma_d/\Sigma_g)$



Numerical simulations of dense midplanes

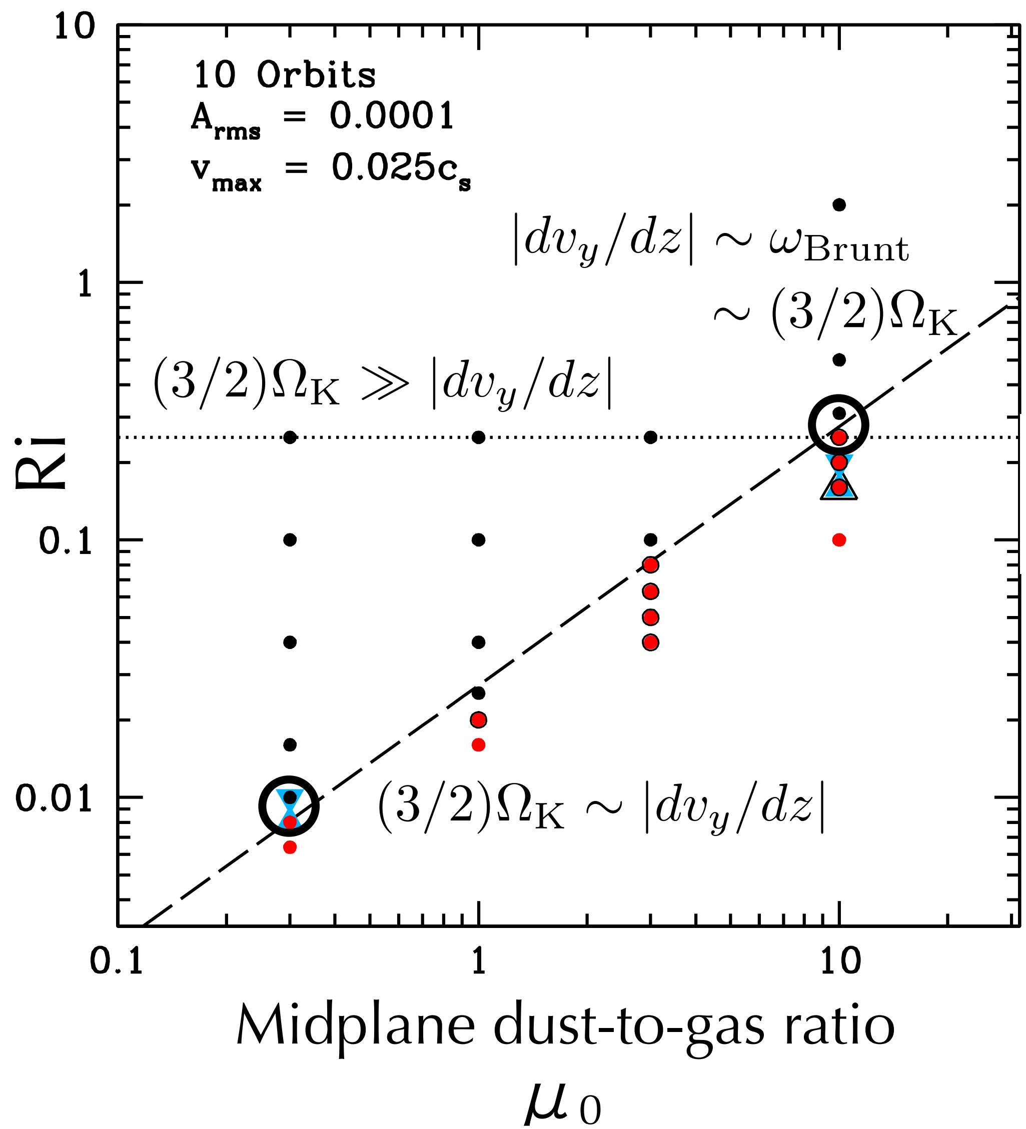


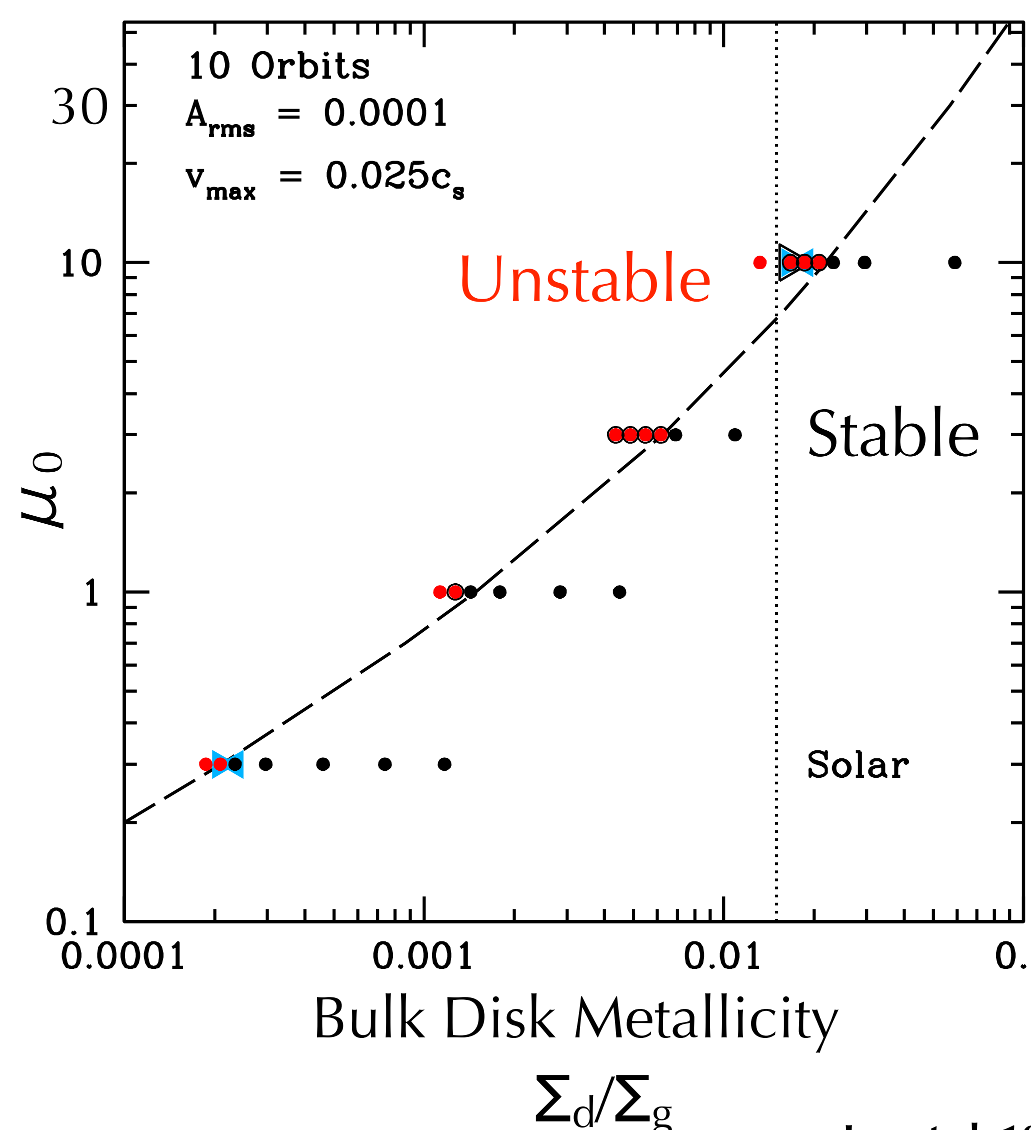




EC 08; Lee, EC, Asay-Davis, and Barranco 10a

Gravitational Instability by Metal Enrichment

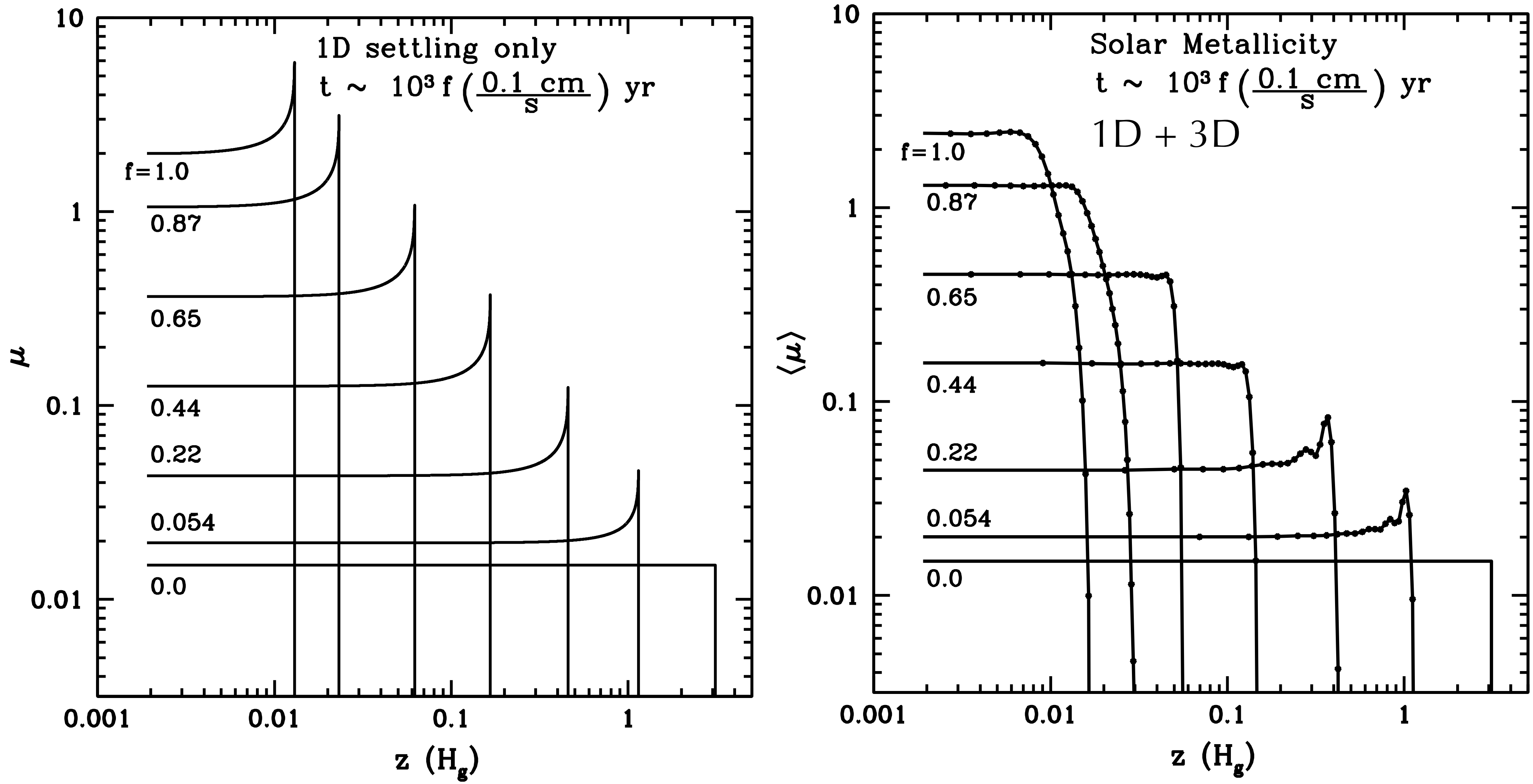




Lee et al. 10a

Toomre-like densities possible for ~4x bulk solar metallicity,
or ~4x more mass than minimum-mass solar nebula

Relaxing constant Ri: Settling from arbitrary initial conditions



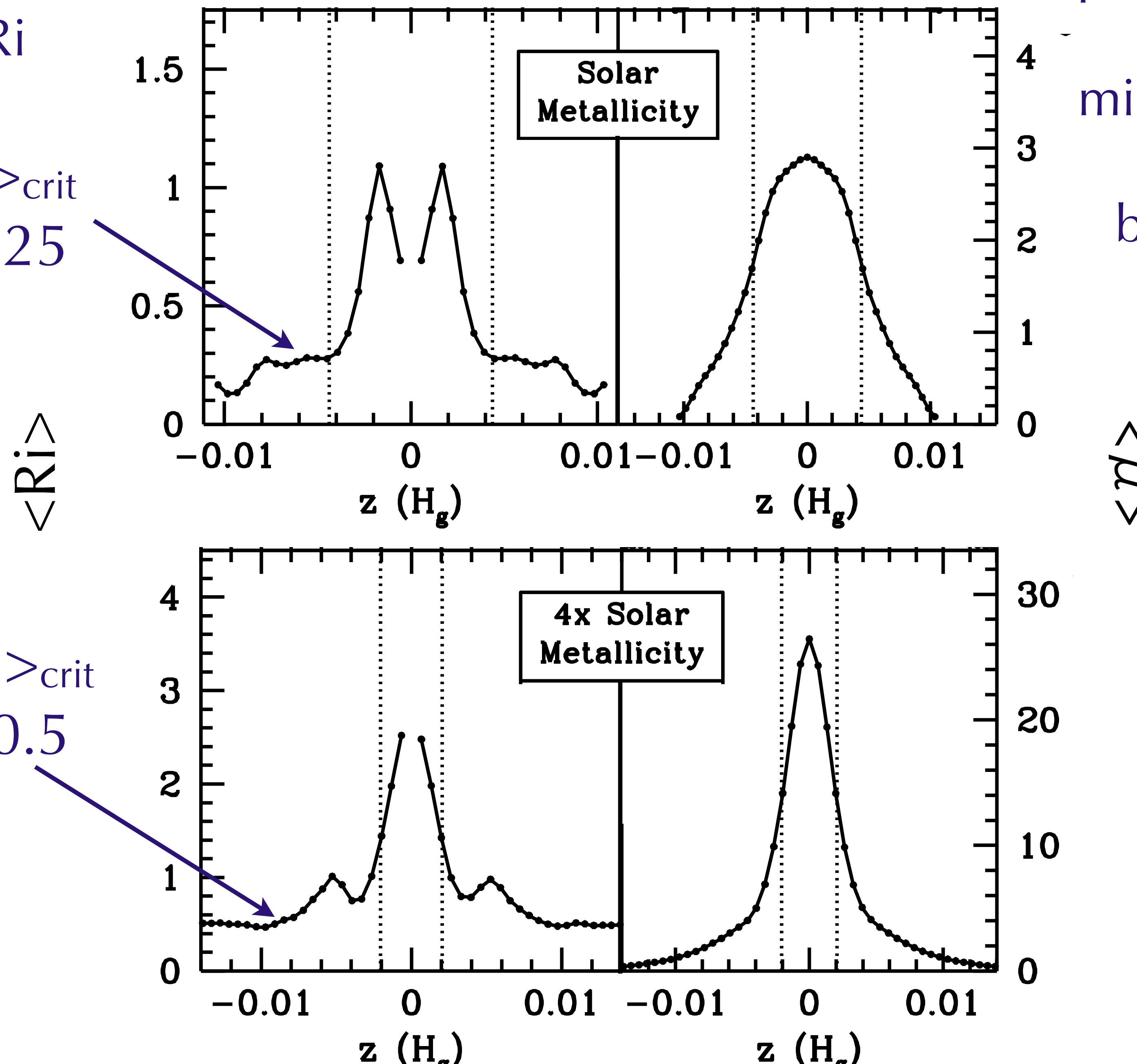
Lee et al. 10b

Finding the marginally stable state

Marginally stable states

Evidence for constant Ri

Superlinear relation between midplane density and bulk metallicity

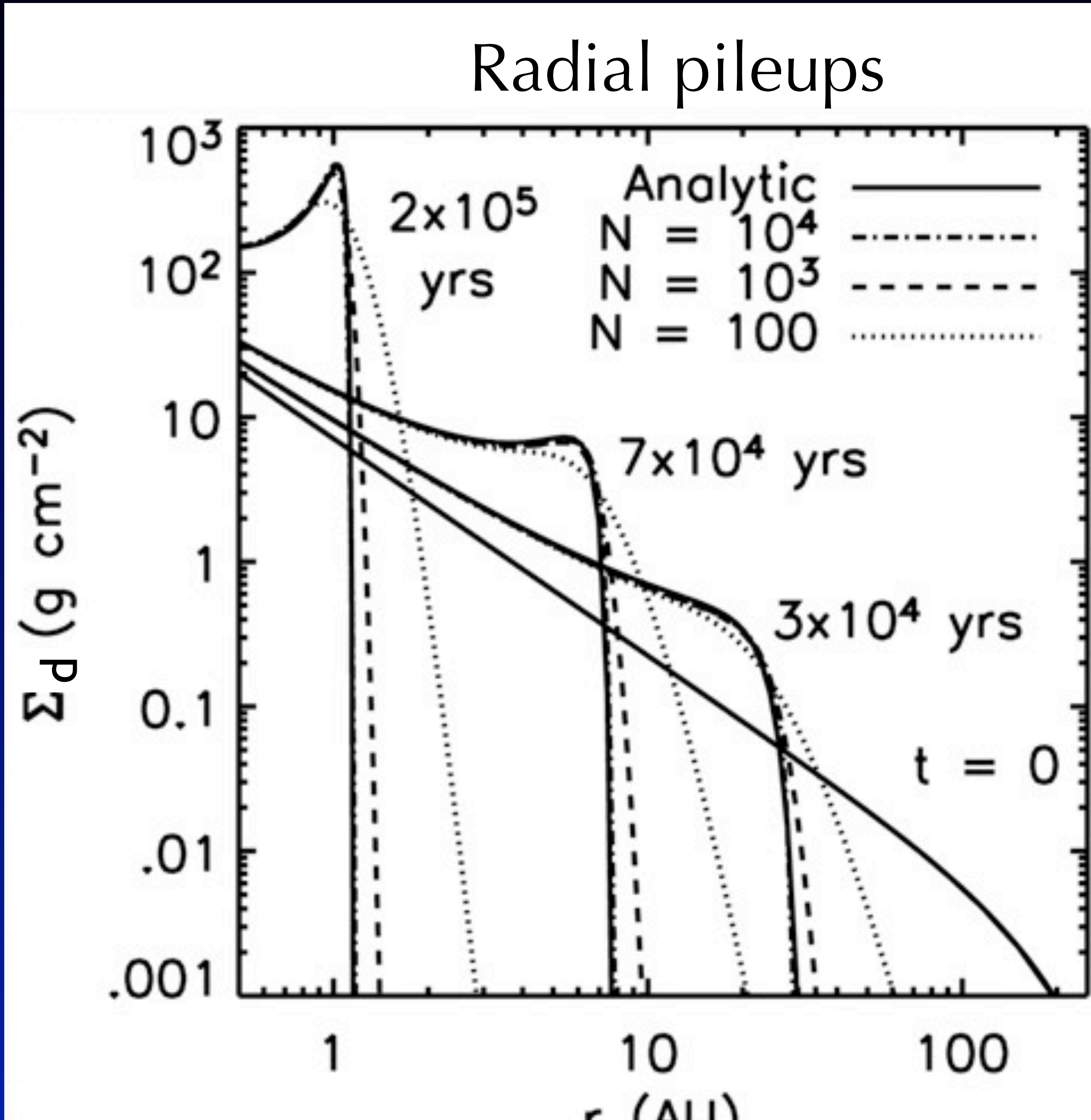


$\langle Ri \rangle_{crit} \approx 0.25$

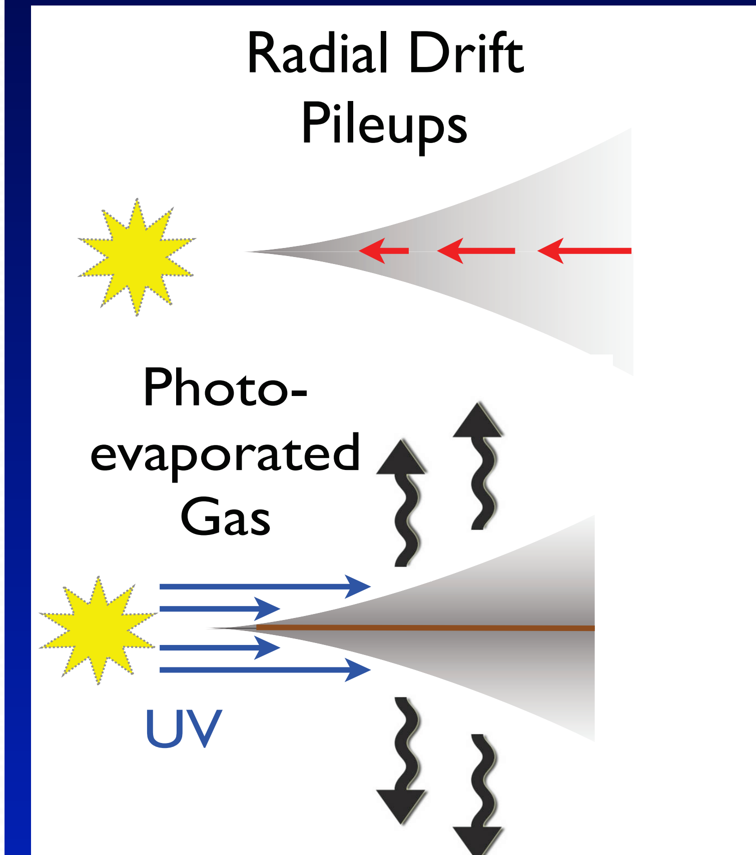
$\langle Ri \rangle_{crit} \approx 0.5$

$\langle \mu_0 \rangle \propto (\Sigma_d/\Sigma_g)^{>1}$

Local enrichments of metallicity



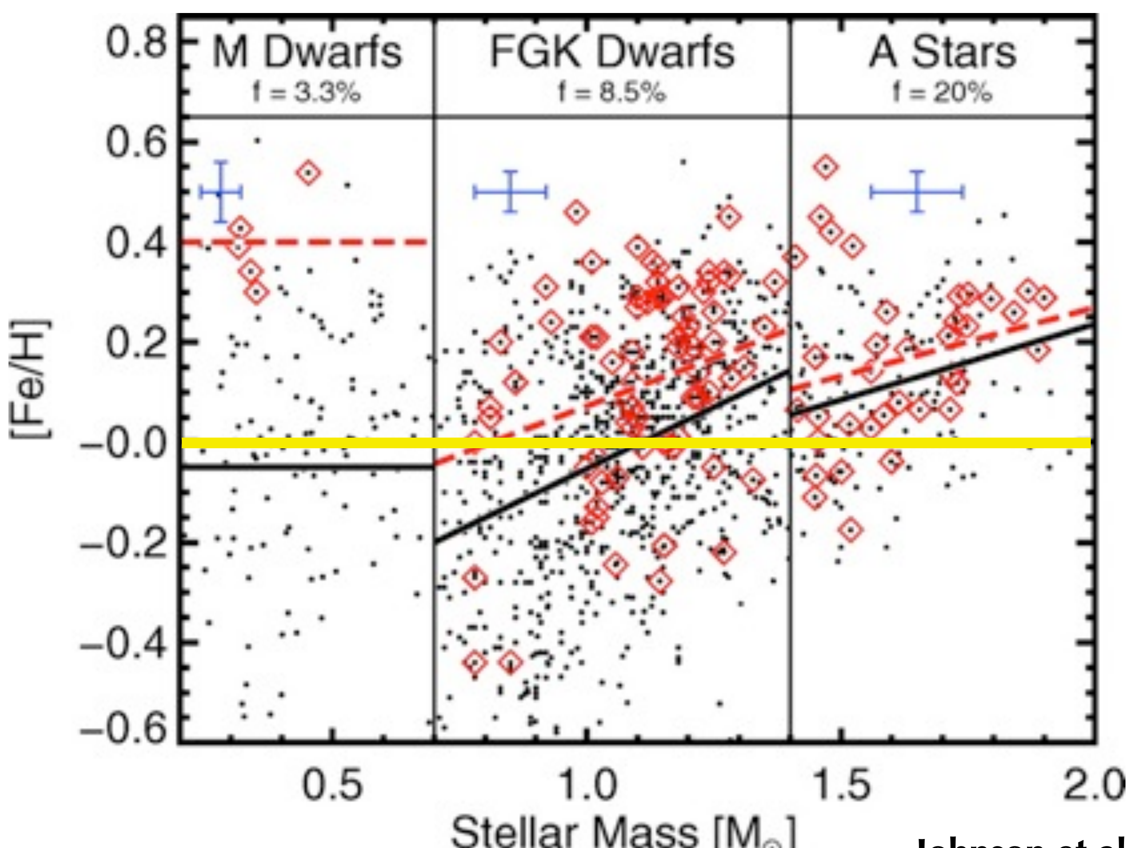
Youdin & EC 04



Modeled metallicities

Guillot et al. 06

Name	M_{planet} ($M_\oplus$)	M_Z[a] ($M_\oplus$)	Z_{planet} (M_Z/M_{planet})	$Z_{planet}/Z_\odot$[b]	$[Fe/H]_*$	Z_{planet}/Z_*
HD209458	210	20	0.095	6.35	0.02	6.06
OGLE-TR-56	394	120	0.304	20.3	0.25	11.418
OGLE-TR-113	429	70	0.163	10.9	0.15	7.7
OGLE-TR-132	350	105	0.3	20	0.37	8.531
OGLE-TR-111	168	50	0.297	19.84	0.19	12.81
OGLE-TR-10	200	10	0.05	3.33	0.28	1.75
TrES-1	238	50	0.21	14.0	0.06	12.2
HD149026	114	80	0.70	46.78	0.36	20.42
HD189733	365	30	0.082	5.479	-0.03	5.87
Jupiter	318	10–42	0.03–0.13	2.0–8.8	0	2.0–8.8
Saturn	95.2	15–30	0.16–0.32	11–21	0	11–21



Johnson et al. 10

Toomre density requires:

Super-solar bulk metallicities

(< 4 x; $\Sigma_d/\Sigma_g = 0.05$)

and/or

Disk masses >

minimum-mass nebula (< 4 x)

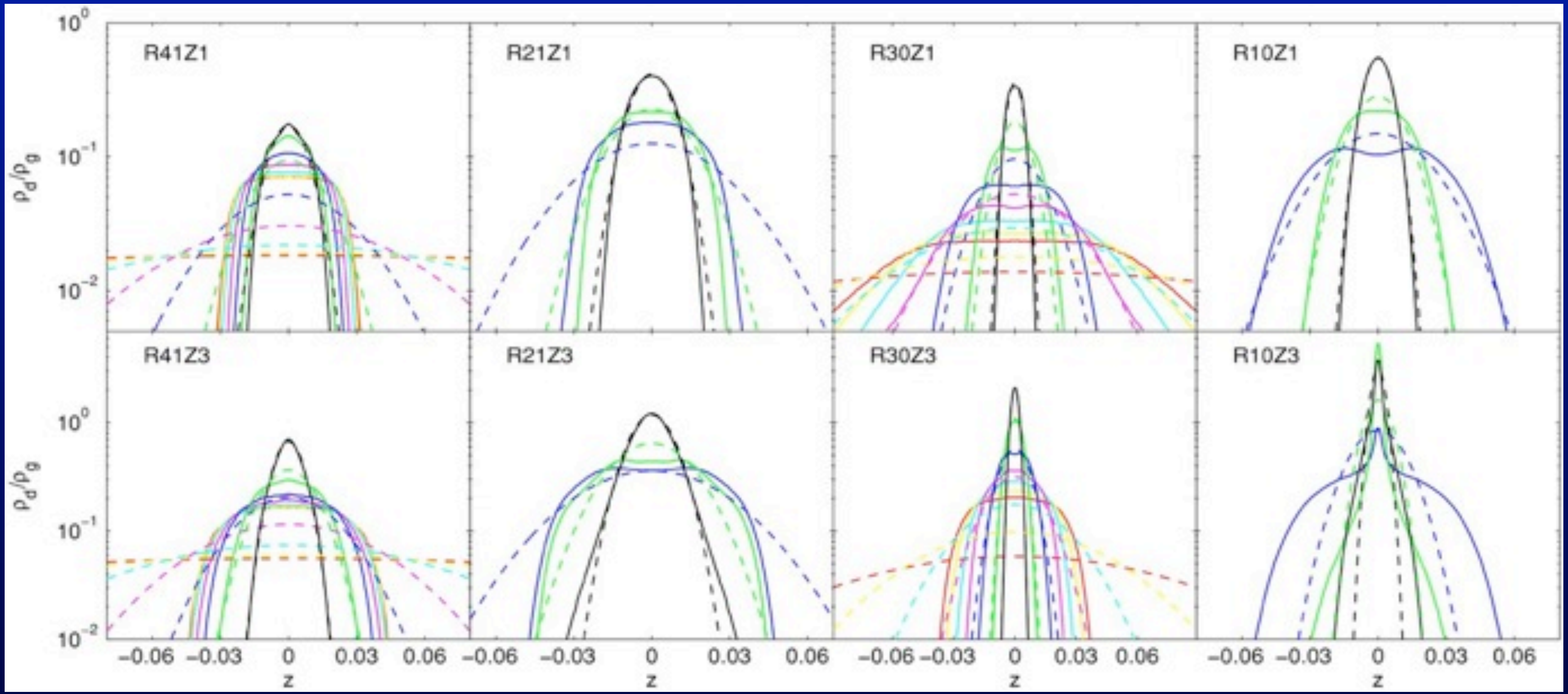
Planetesimal Formation

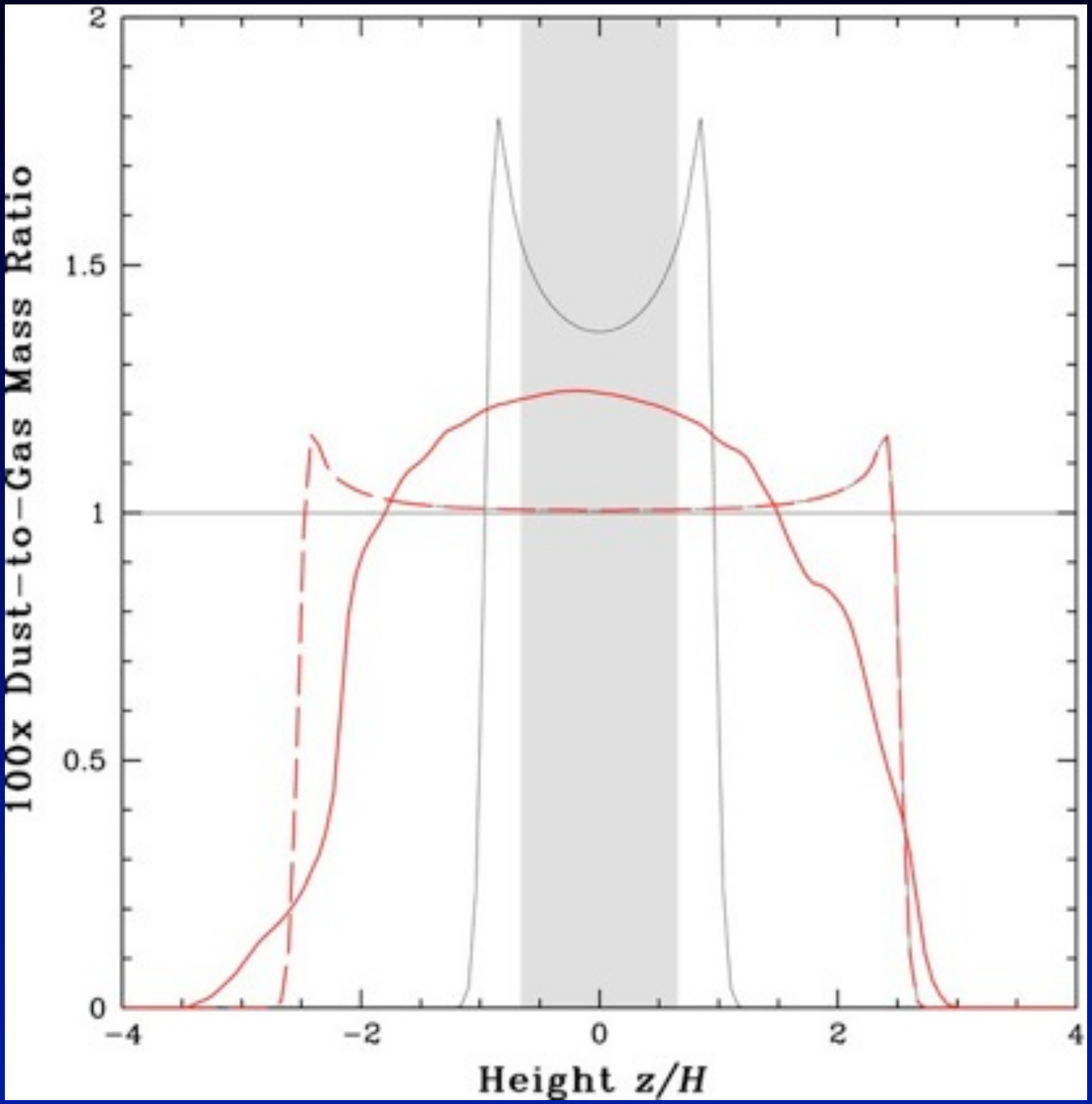
EC (Berkeley)
Aaron Lee (Berkeley)
Joe Barranco (SFSU)
Xylar Asay-Davis (LANL)

Can dust in disk midplanes become self-gravitating?

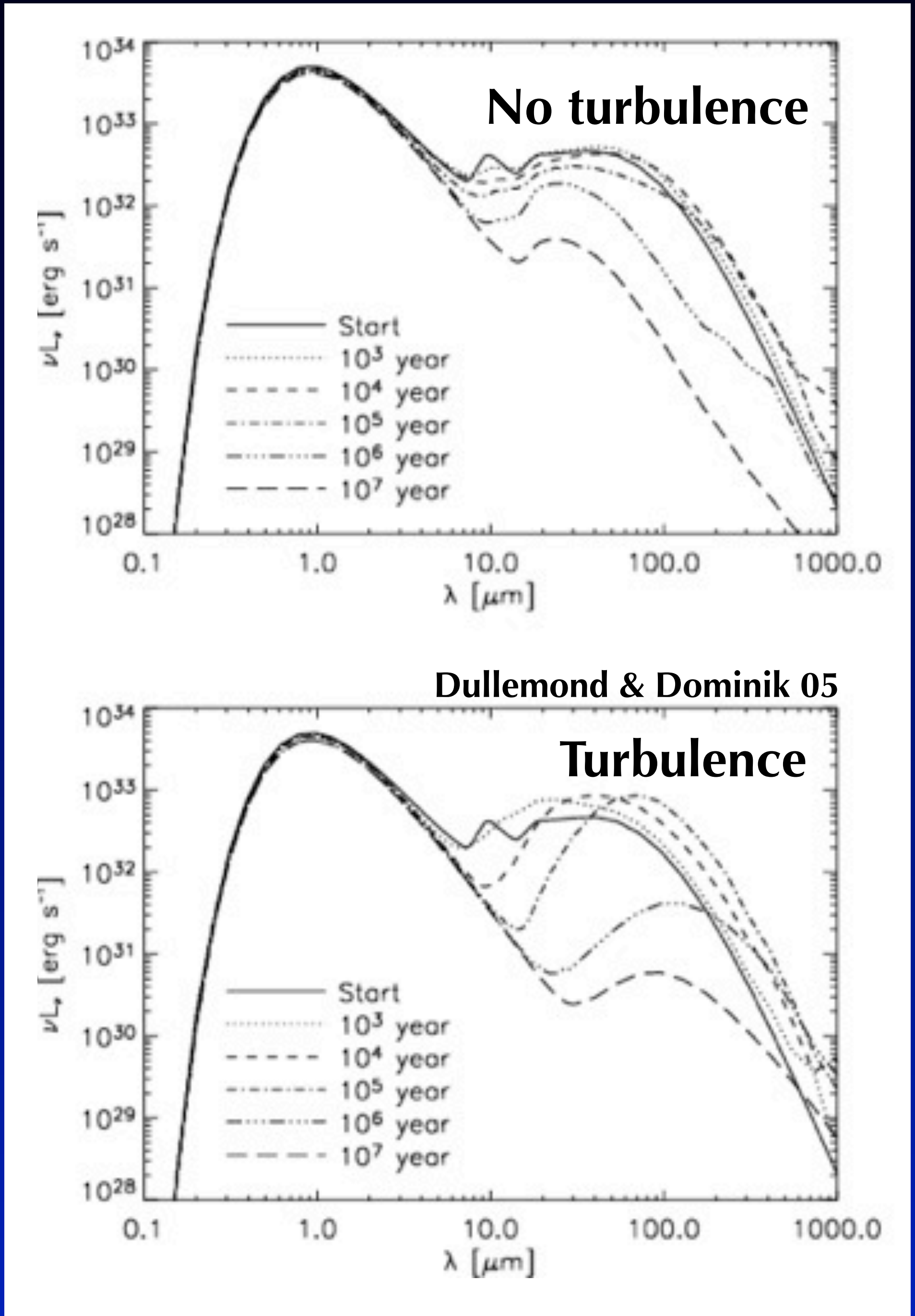
Yes, if local bulk metallicity is a few times solar,
or if local surface density is a few times minimum-mass solar nebula

Extra Slides


R41Z1
R21Z1
R30Z1
R10Z1
R41Z3
R21Z3
R30Z3
R10Z3
ρ_d/ρ_g
10^0
10^{-1}
10^{-2}
−0.06
−0.03
0
0.03
0.06
z


100x Dust-to-Gas Mass Ratio
2
1.5
1
0.5
0
−4
−2
0
2
4
Height z/H

Theory I: Grain growth: Right sign, wrong magnitude



Sticks too well

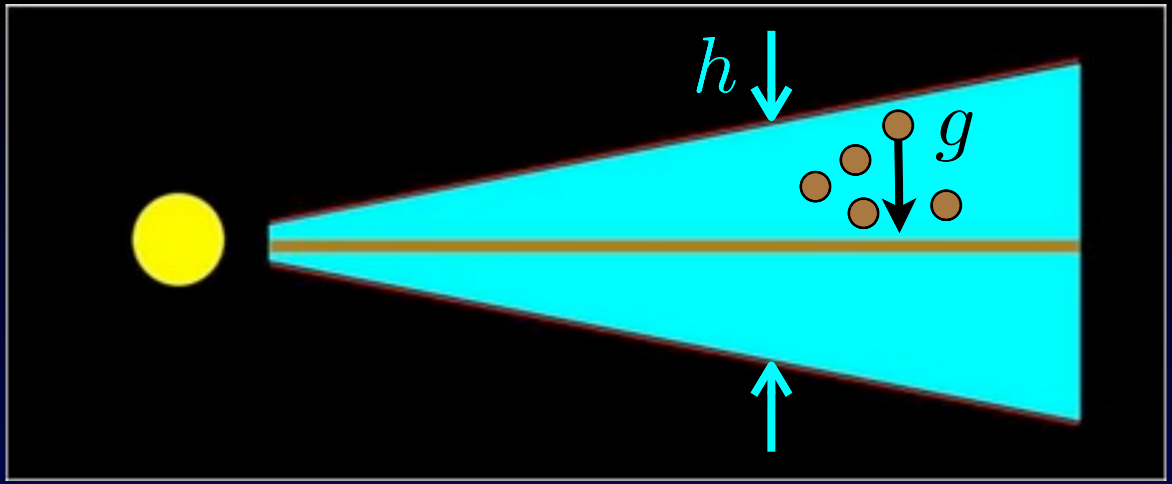
Problem persists even if

- grains are fractal
- monomers are nonspherical

Proposed solution:

Replenishment of micron-sized grains (near-IR opacity) by fragmentation

Grain growth



$$mg \sim F_{\rm drag}(v)$$

$$\mu s^3 \Omega^2 h \sim \rho_g s^2 c_s v$$

$\longrightarrow$ Terminal $v \sim \dfrac{\mu}{\rho_g}\Omega s$ (bigger is faster)

Accretion $\dfrac{d}{dt}(\mu s^3) \sim \rho_d v s^2 \longrightarrow \dot{s} \sim \dfrac{\rho_d}{\mu} v$ (faster is bigger)

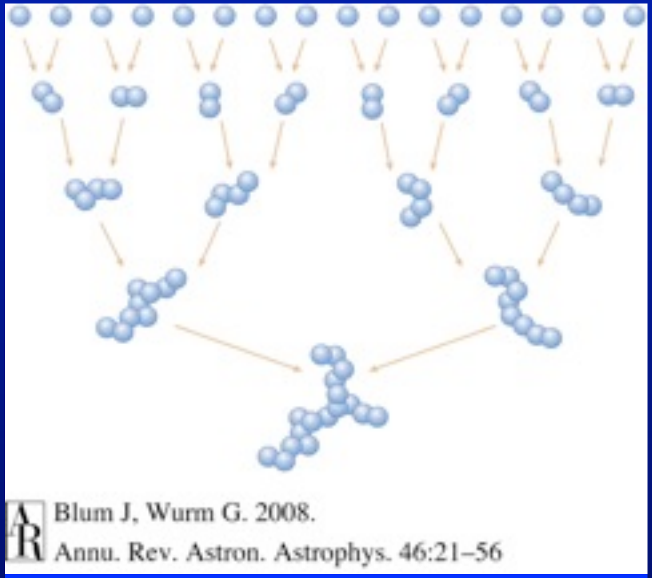
$\longrightarrow$ Exponential growth $s \sim s_0 \exp(\rho_d \Omega t/\rho_g)$ (fastest growth in inner disk)

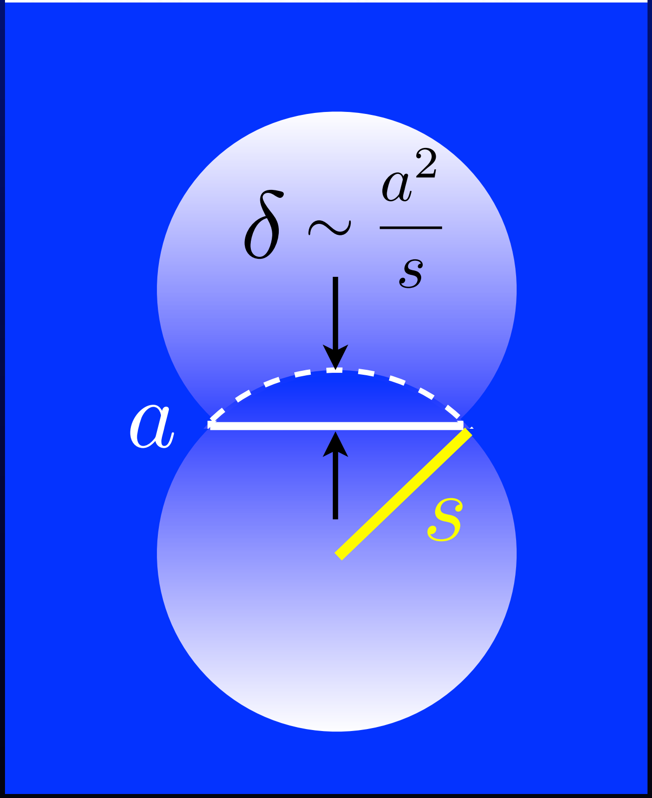
Since $t \sim h/v \longrightarrow s \sim s_0 \exp(\Sigma_d/\mu s)$

$$\left.\begin{array}{r} s_0 \sim 1\,\mu\mathrm{m} \\ \mu \sim 1\,\mathrm{g\,cm^{-3}} \\ \Sigma_d \sim 10\,\mathrm{g\,cm^{-2}} \end{array}\right\} \begin{array}{l} s \sim 1\,\mathrm{cm} \\ t \sim 100\,\mathrm{yr} \\ v \sim 1\,\mathrm{m/s} \end{array}$$

Safronov 1969

Grain growth





$$\boxed{v_{\rm crit} \sim 1\,{\rm m/s}\ {\rm for}\ s \sim 1\,\mu{\rm m}}$$

Repulsion (elastic modulus E)

$$\text{Stress}\,\sigma \sim E\nabla\xi \sim E\frac{\delta}{a}$$

$$\sigma \sim \frac{mv}{(\delta/v)\times a^2}$$

$$\text{Repulsive force}\,F_R \sim \sigma a^2 \sim \mu^{3/5}E^{2/5}s^2v^{6/5}$$

$$\text{Repulsive energy}\,U_R \sim F_R\delta$$

Adhesion (surface tension γ)

$$\text{Binding energy}\,U_B \sim \gamma a^2$$

$$U_R = U_B \longrightarrow v_{\rm crit} \sim 4\frac{\gamma^{5/6}}{E^{1/3}\mu^{1/2}s^{5/6}}$$

Hertz 1882, Chokshi et al. 93

Rotational Effects

Ri

Coriolis	Kepler radial shear	Brunt oscillation	Vertical shear
2Ω	$\frac{3}{2}\Omega$	$< \Omega$	$< \Omega$
destabilizing Cabot 84 Gomez & Ostriker 05	stabilizing Ishitsu & Sekiya 03 EC 08 Barranco 09	stabilizing	destabilizing