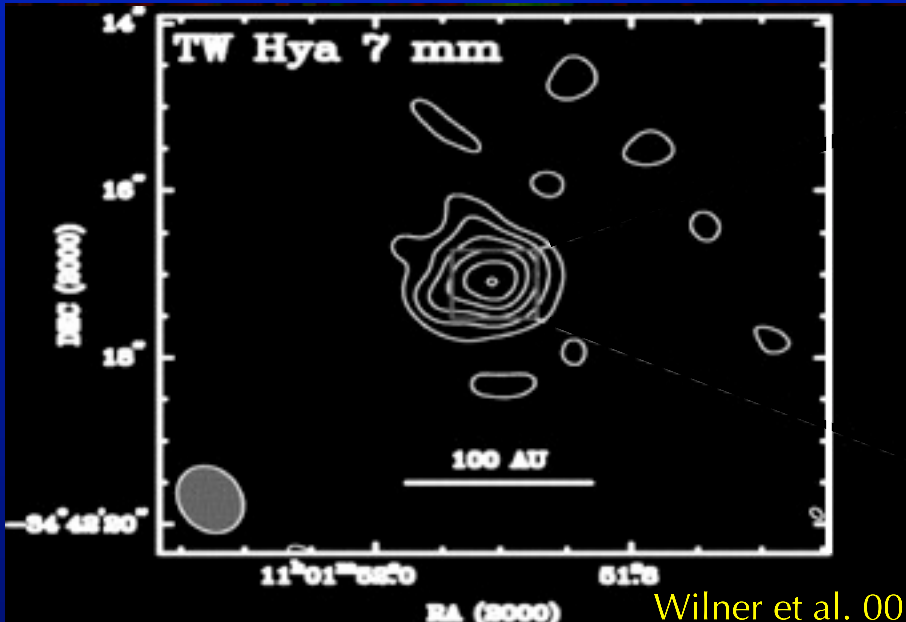
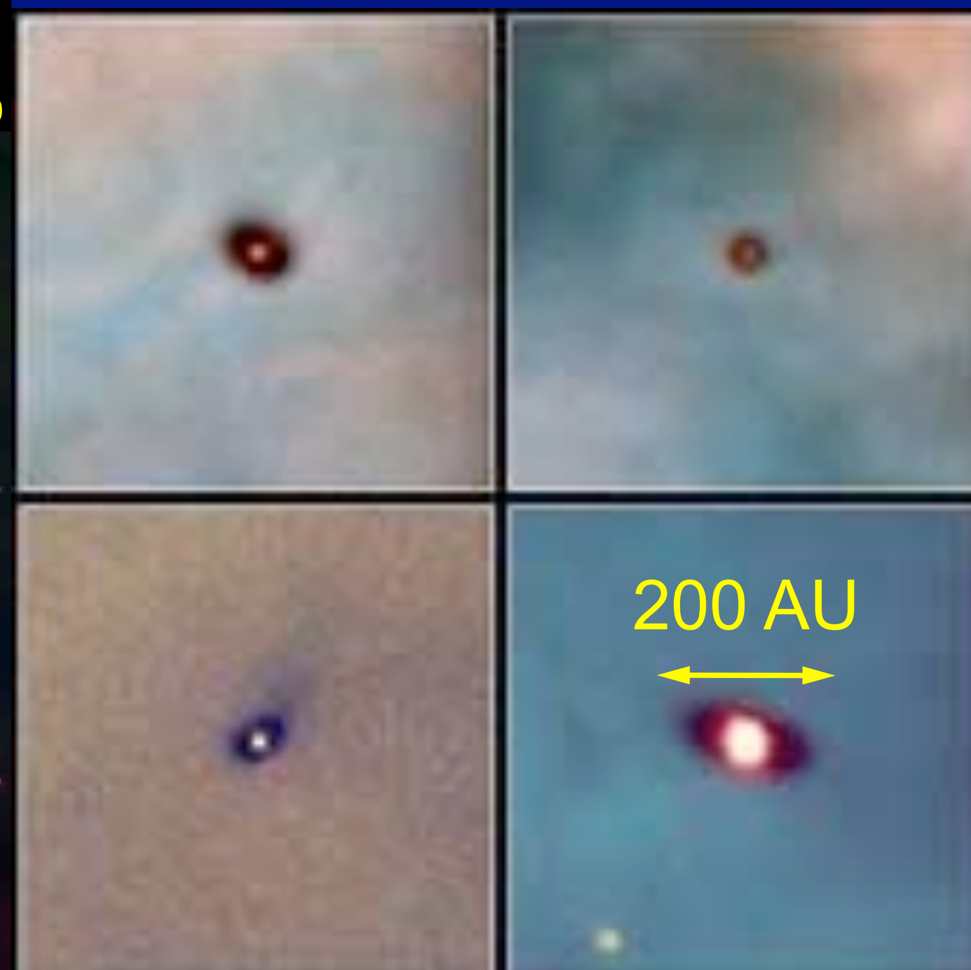
# Planetesimal Formation and Planet Coagulation

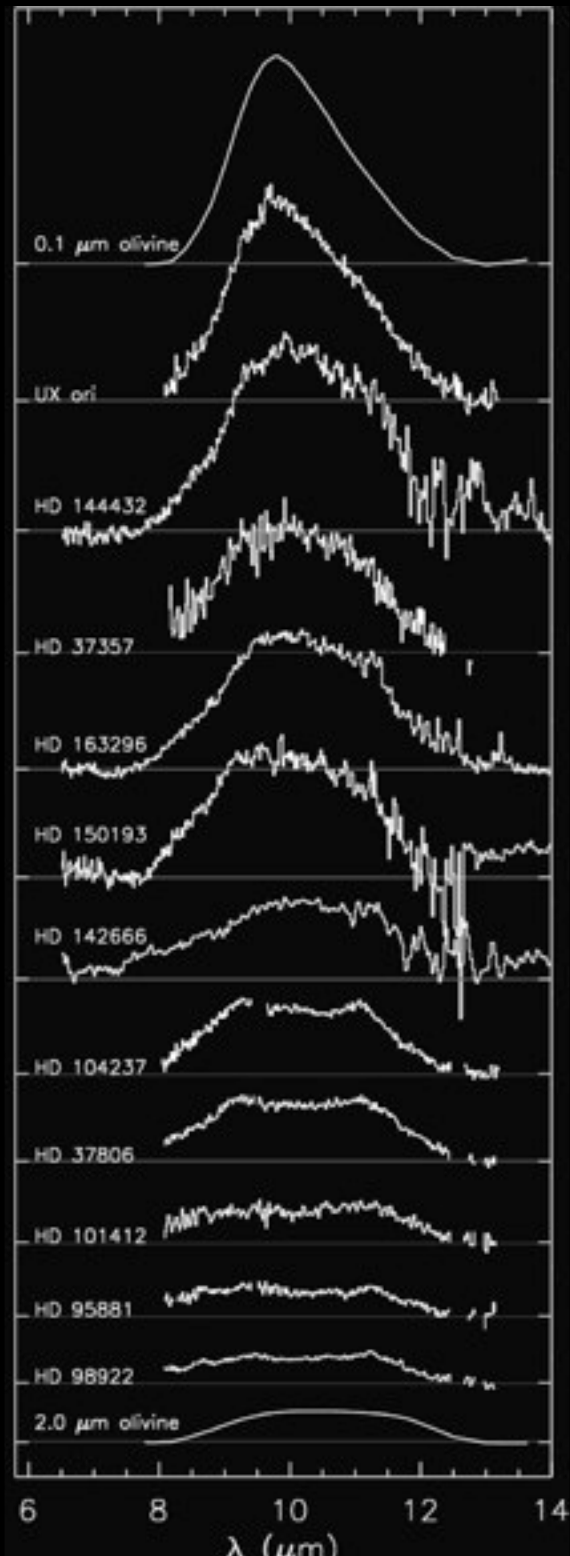
# Protoplanetary Disks

disk mass ~ 0.001-0.1 stellar mass




Wilner et al. 00





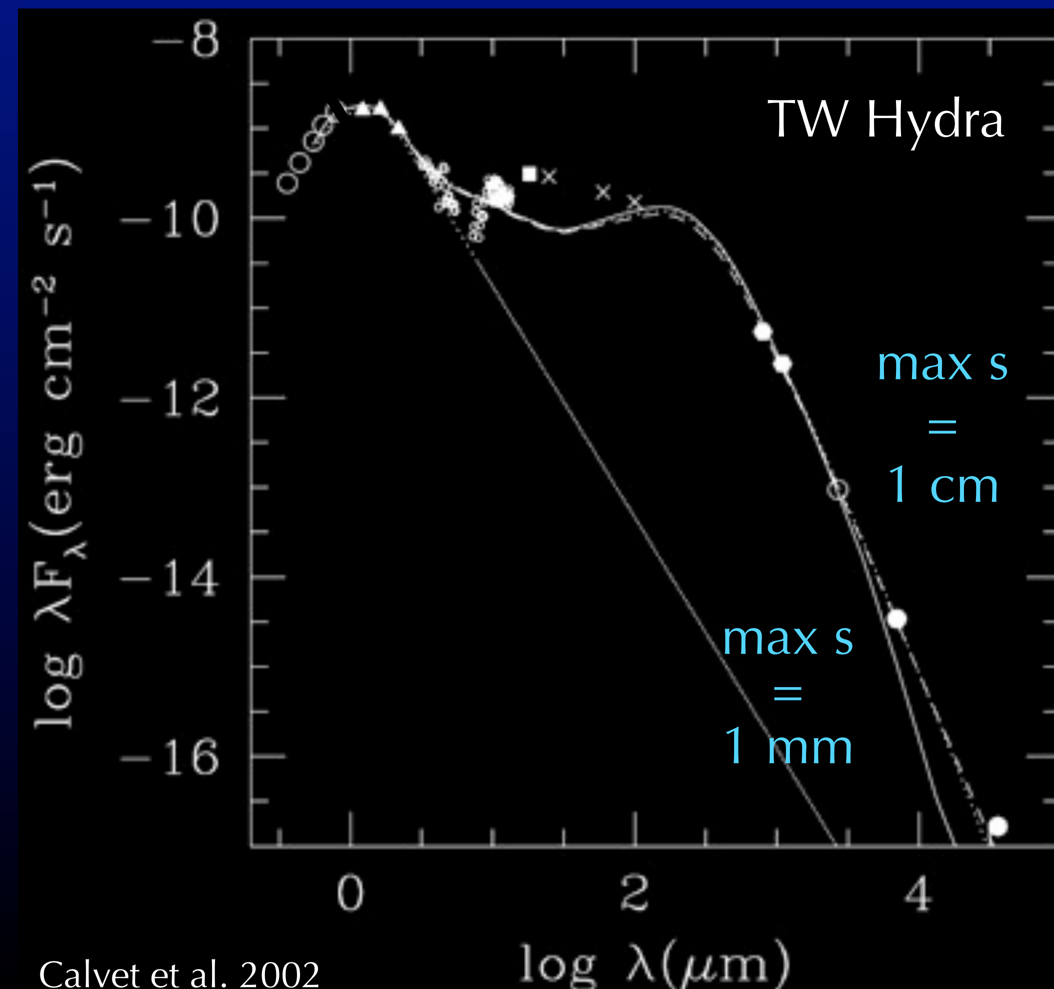

van Boekel et al. 03

$F_\nu$

# Disk surfaces at ~10 AU: Growth to a few microns

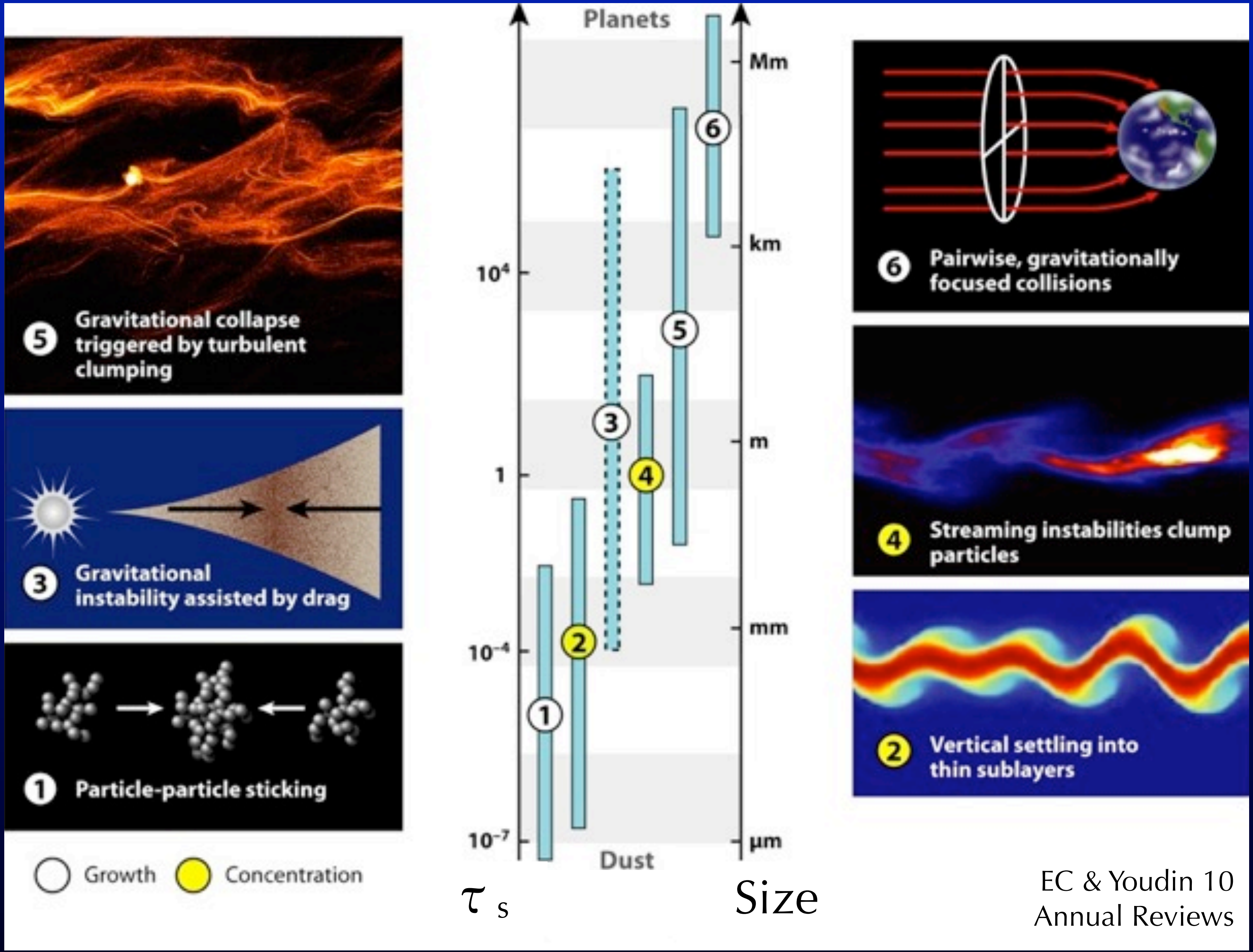

McCabe, Duchene, & Ghez 03

# Disk interior at ~100 AU: Growth to a few cm




Calvet et al. 2002

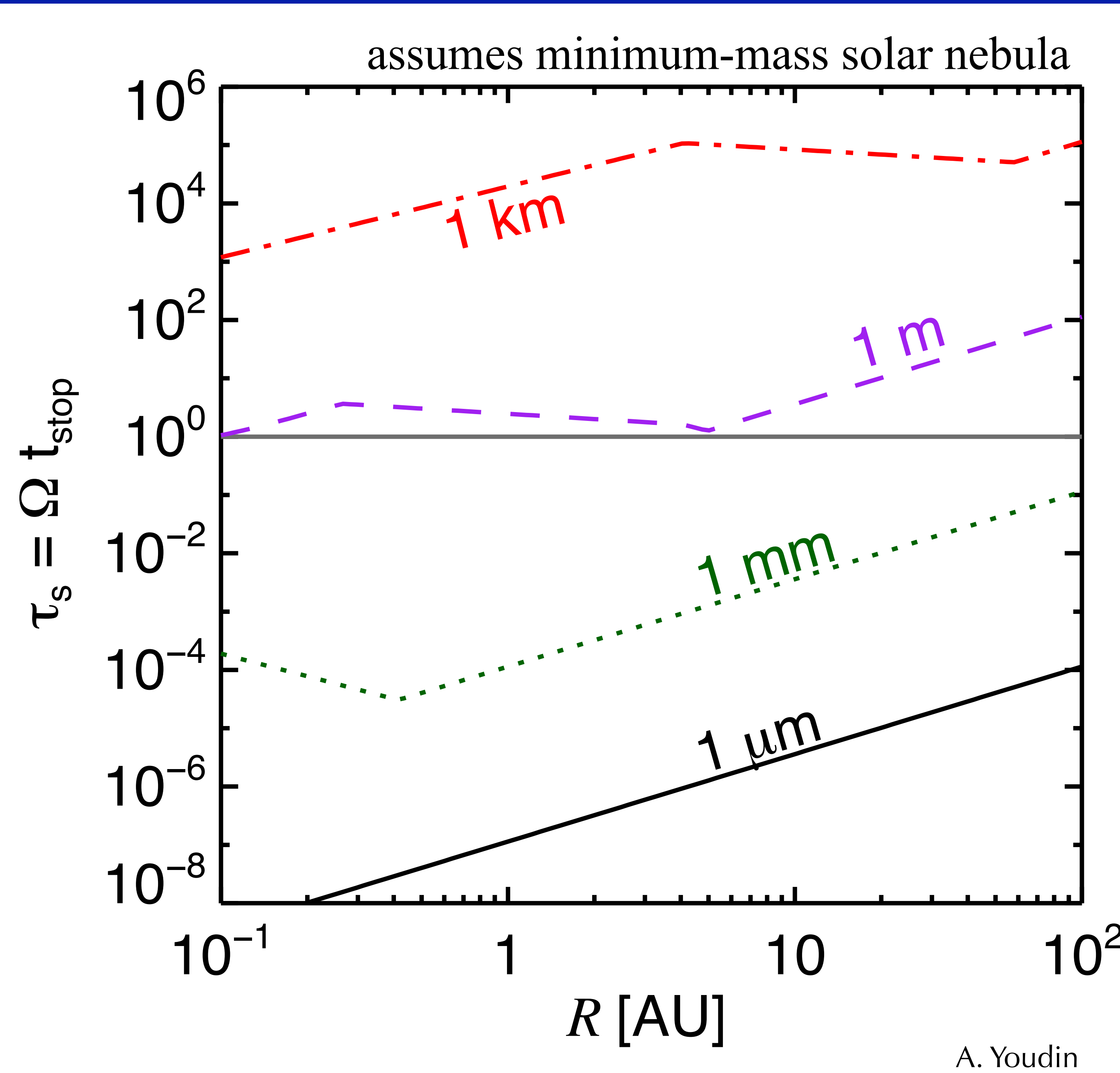
# Climbing the size ladder




Grain stopping time $t_{stop} \equiv m v_{rel} / F_{drag}$

Dimensionless stopping time $\tau_s \equiv \Omega_{Kepler} t_{stop}$

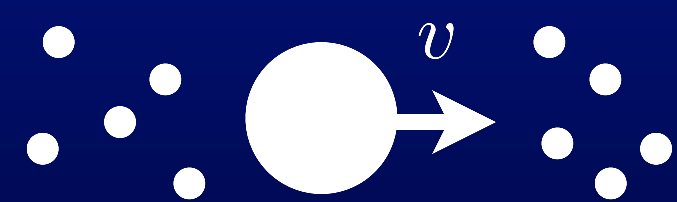
# Gas-particle entrainment




e.g., Epstein drag

$$\lambda_{\rm mfp} > s$$

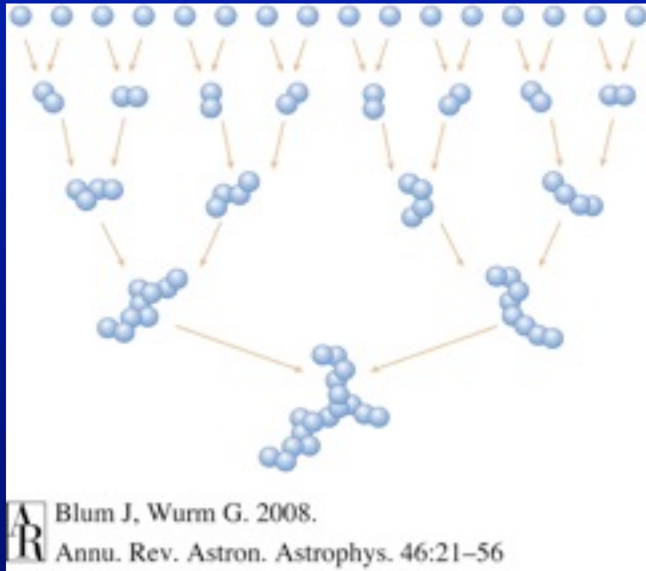
$$v < c_s$$

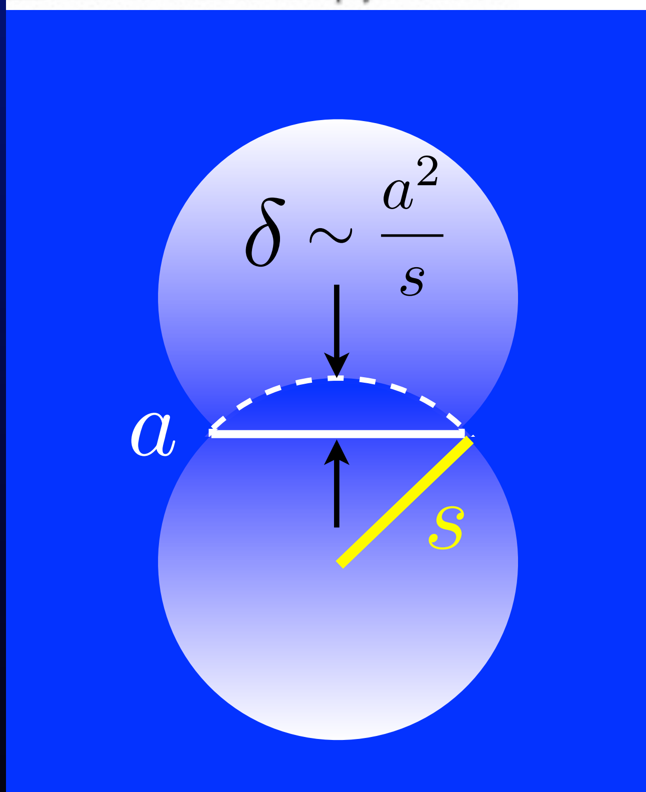


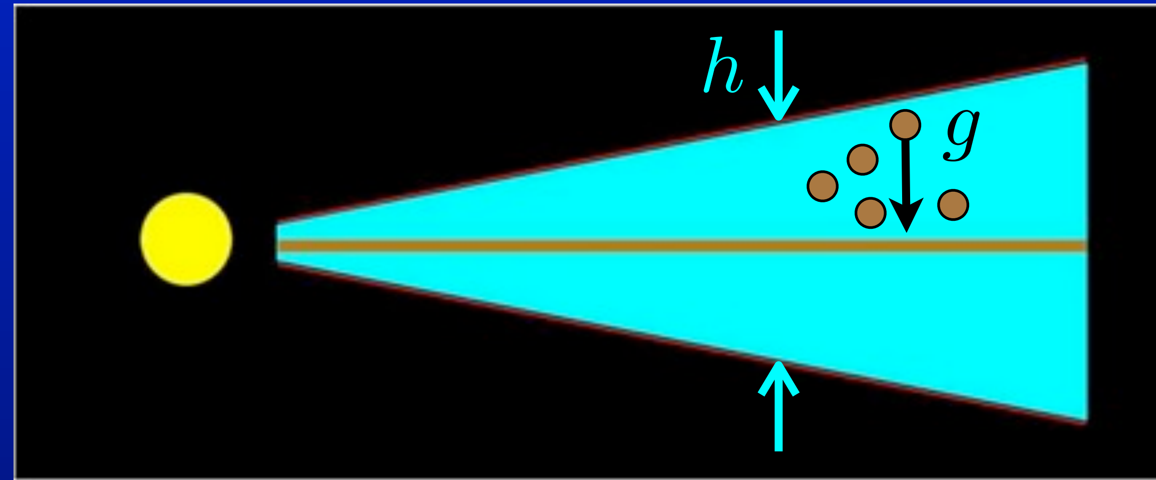

$\rho(c_s - v)^2$ $\qquad$ $\rho(c_s + v)^2$

$$\Rightarrow F_{\rm drag} \sim \rho c_s v \times \pi s^2$$

# Grain growth




Sticking $v_{\rm stick} \sim 1\,{\rm m/s}$ for $s \sim 1\,\mu{\rm m}$

Hertz + surface tension $\longrightarrow v_{\rm stick} \sim 4\frac{\gamma^{5/6}}{E^{1/3}\rho^{1/2}s^{5/6}}$

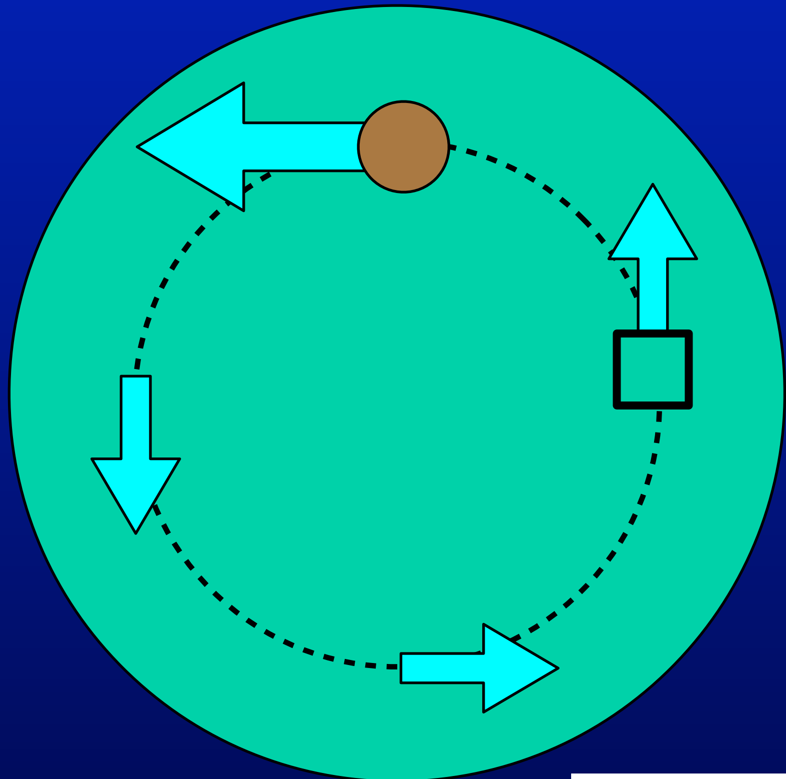
Terminal $v_{\rm term} \sim \frac{\rho}{\rho_g}\Omega s$

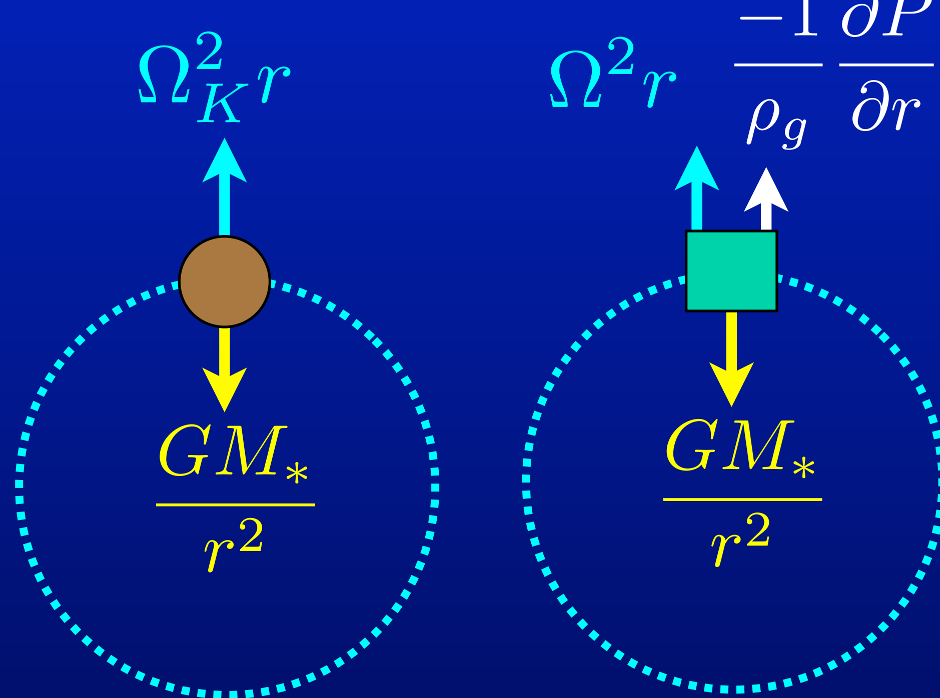
$\sim 1\,{\rm m/s}$ for $s \sim 10\,{\rm cm}$

Sticking up to, but not beyond, cm sizes

Chokshi et al. 93, Blum & Wurm 08

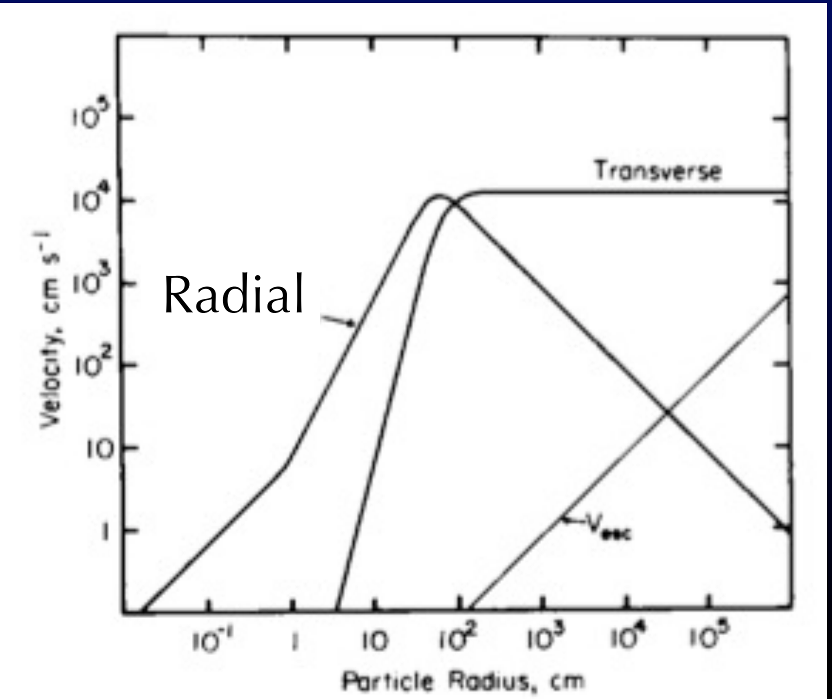
# Radial drift from headwind






$\therefore\ \Omega < \Omega_K$

Meter-sized boulders drift inward from 1 AU within 100 yr




Weidenschilling 77
Nakagawa, Sekiya, & Hayashi 86

# Gravitational instability

Disk annulus can start to fragment if Toomre Q ~ 1

$$\langle \rho \rangle > \rho_{\rm Toomre} \sim \frac{M_*}{2\pi r^3}$$

$$> 10^{-7}\,{\rm g\,cm^{-3}}$$

Clump can resist tidal shear if Roche unstable

$$\rho > \rho_{\rm Roche} \sim \frac{3.5 M_*}{r^3}$$

$$> 2 \times 10^{-6}\,{\rm g\,cm^{-3}}$$

$$\text{if } \Sigma_g \sim 2000\,{\rm g\,cm^{-2}}\ ({\rm minimum-mass\,solar\,nebula})$$

$$\text{if } \Sigma_d/\Sigma_g \sim 10^{-2}\ ({\rm height-integrated\,solar\,metallicity})$$

$$\text{then } \rho \sim \frac{\Sigma_g}{h_g} + \frac{\Sigma_d}{h_d}$$

$$\sim 3 \times 10^{-9}{\rm g\ cm^{-3}} \qquad \text{if } h_d \sim h_g$$

<u>Toomre unstable</u>

$$\rho_d/\rho_g \sim 30$$

Toomre mass $\sim \rho_{\rm Toomre} \lambda^3_{\rm unstable}$

$$\sim 10^{19}\,{\rm g}\ (10\,{\rm km})$$

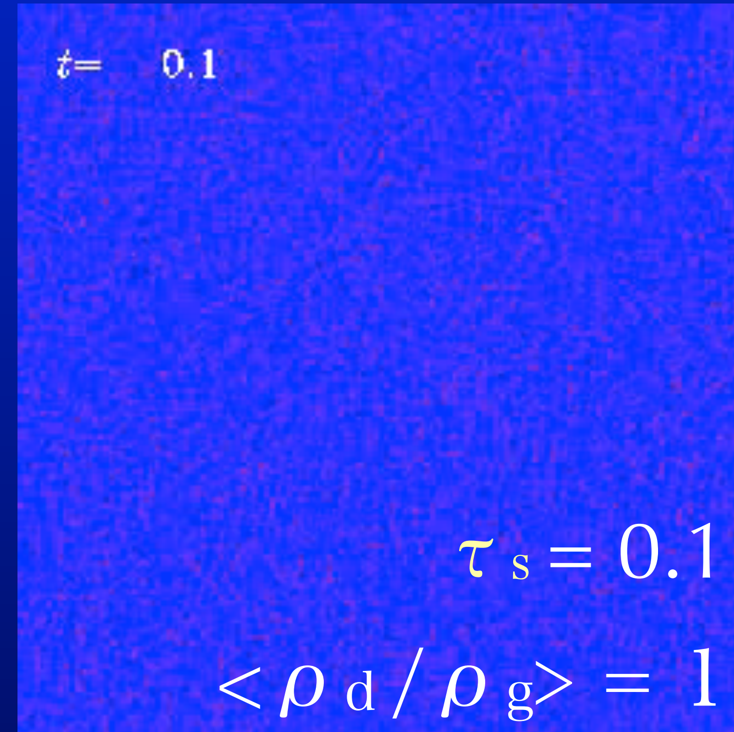
<u>Roche unstable</u>

$$\rho_d/\rho_g \sim 600$$

"Streaming" instability
= linear instability between
two fluids interacting
frictionally in a disk

growth rates peak
for $\tau_s \sim 1$

(marginally coupled bodies)



Youdin & Goodman 05; Johansen & Youdin 07

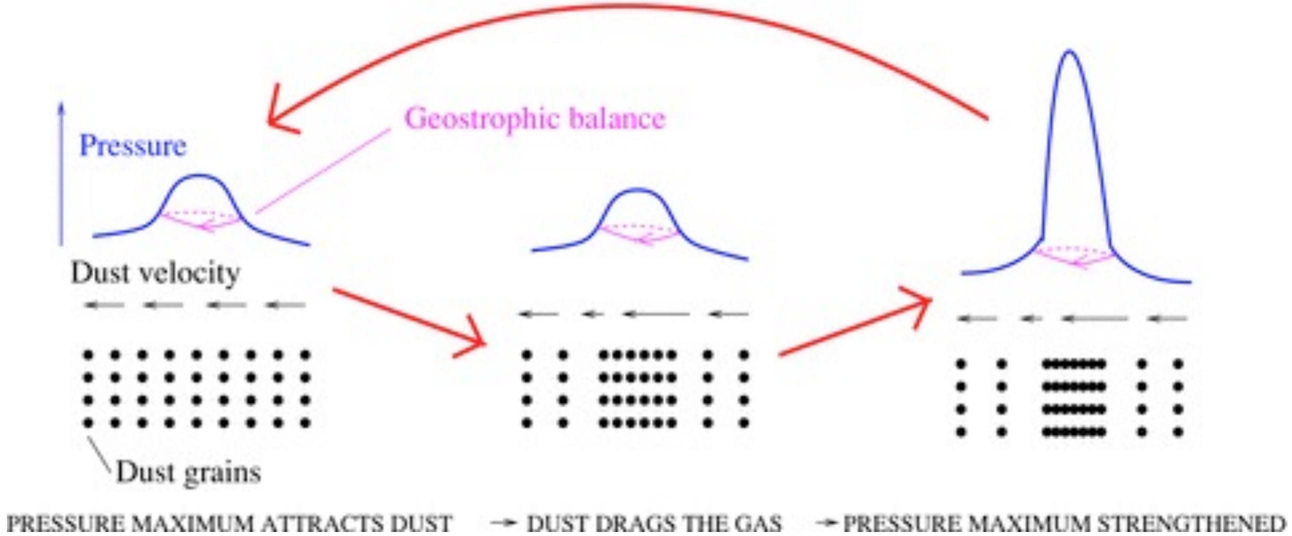
$$\nabla \cdot \boldsymbol{v}_g = 0,$$

$$\frac{D_p \rho_p}{Dt} = -\rho_p \nabla \cdot \boldsymbol{v}_p,$$

$$\frac{D_p \boldsymbol{v}_p}{Dt} = -\Omega_K^2 \boldsymbol{r} - \frac{\boldsymbol{v}_p - \boldsymbol{v}_g}{t_s},$$

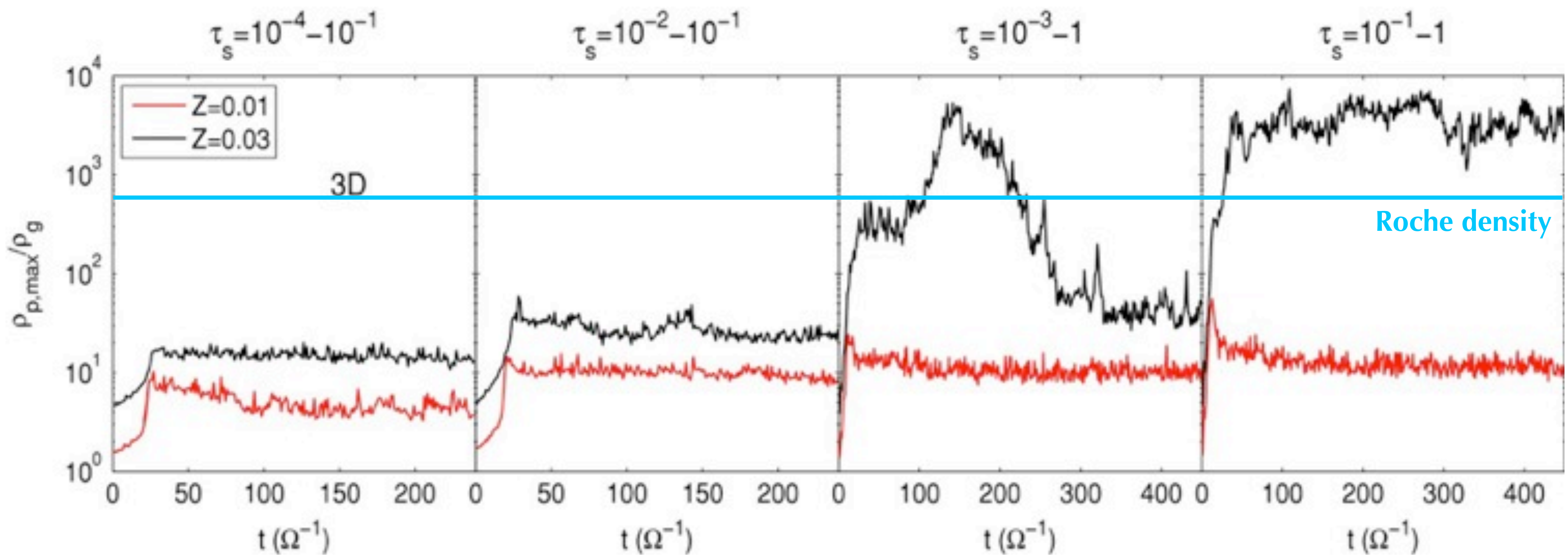
$$\frac{D_g \boldsymbol{v}_g}{Dt} = -\Omega_K^2 \boldsymbol{r} + \frac{\rho_p}{\rho_g} \frac{\boldsymbol{v}_p - \boldsymbol{v}_g}{t_s} - \frac{\nabla P}{\rho_g}$$

# Streaming instability: Physical interpretation




Jacquet et al. 2011

# Relies strongly on marginally coupled bodies




Bai & Stone 2011

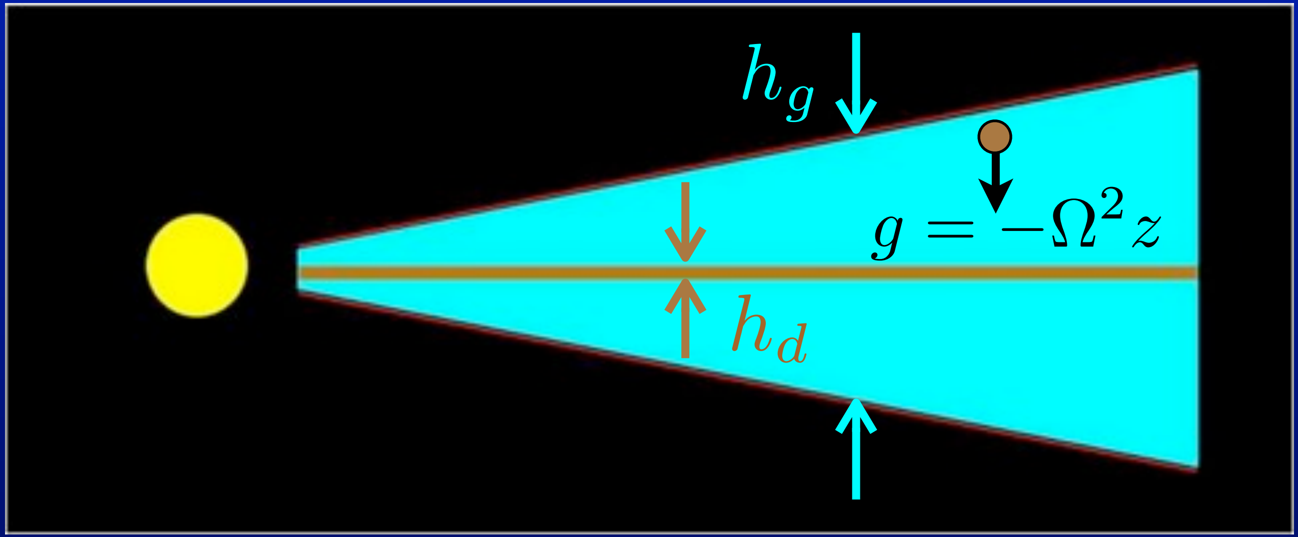
# Gravitational instability in the $\tau_s << 1$ (small particle) limit

Self-gravity important when

$$\rho > \rho_{\text{Toomre}} \sim \frac{M_*}{2\pi r^3}$$

$$> 10^{-7}\,\text{g}\,\text{cm}^{-3}$$

$$\text{at } r = 1\,\text{AU}$$




$$\text{if } \Sigma_g \sim 2000\,\text{g}\,\text{cm}^{-2}\ (\text{minimum} - \text{mass solar nebula})$$

$$\text{if } \Sigma_d/\Sigma_g \sim 10^{-2}\ (\text{height} - \text{integrated solar metallicity})$$

$$\text{then } \rho \sim \frac{\Sigma_g}{h_g} + \frac{\Sigma_d}{h_d}$$

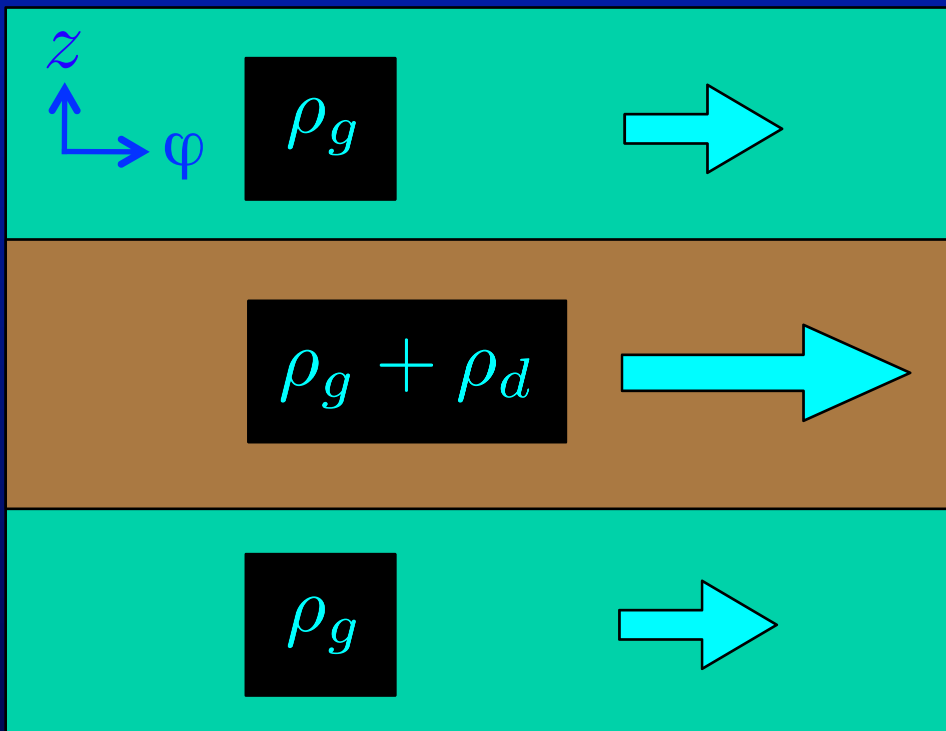
$$\sim 3 \times 10^{-9}\text{g cm}^{-3} \qquad \text{if } h_d \sim h_g$$

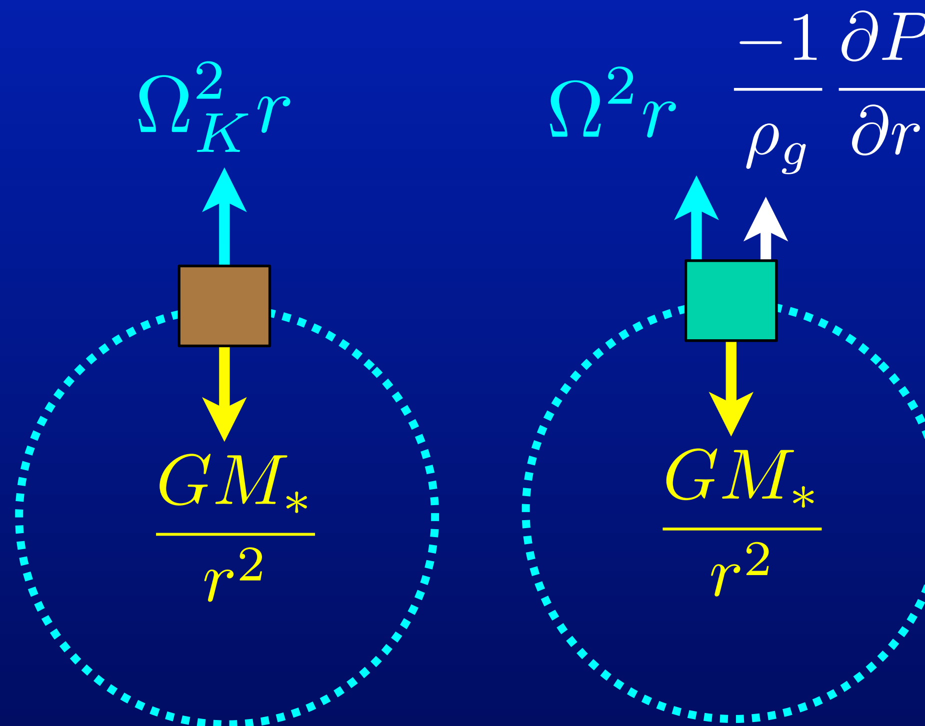
$$\sim 10^{-7}\,\text{g}\,\text{cm}^{-3} \qquad \text{if } h_d \sim 5 \times 10^{-4} h_g$$

Can the settled "sublayer" achieve
Toomre density $< \rho_d / \rho_g > \sim 30$?

Safronov 1969
Goldreich & Ward 1973

# Kelvin-Helmholtz instability may limit dust settling




$$\therefore\ \Omega < \Omega_K$$

$$\Delta v \sim c_s \frac{c_s}{v_{\mathrm{K}}} \sim 25\,\mathrm{m/s}\ \text{ nearly independent of } r$$

Weidenschilling 1980

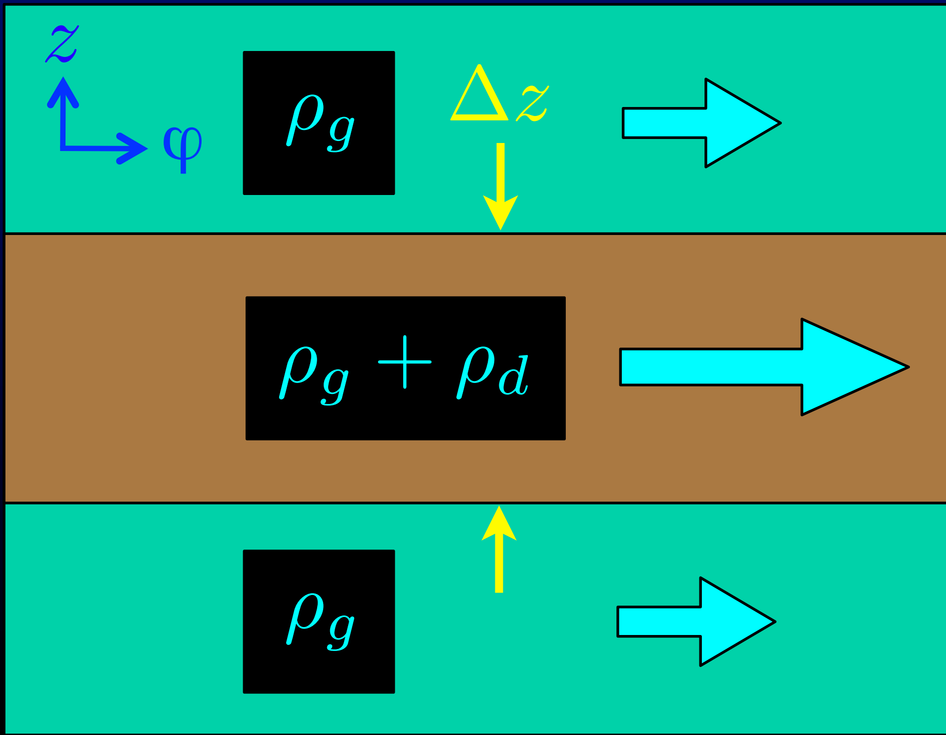
Necessary criterion for K-H instability in Cartesian shear flow:

$$\text{Richardson} \quad Ri \equiv \frac{g\,\partial \ln \rho / \partial z}{(\partial v/\partial z)^2} < Ri_{\rm crit} = \frac{1}{4}$$

$$= \frac{\omega^2_{Brunt}}{(\partial v/\partial z)^2}$$

$$\propto \frac{1/\Delta z}{(1/\Delta z)^2}$$

$$\propto \Delta z$$



# Numerical simulations of dense midplanes

Initial conditions:
Spatially constant Ri

$$\mu(z) \equiv \frac{\rho_d}{\rho_g}(z)$$

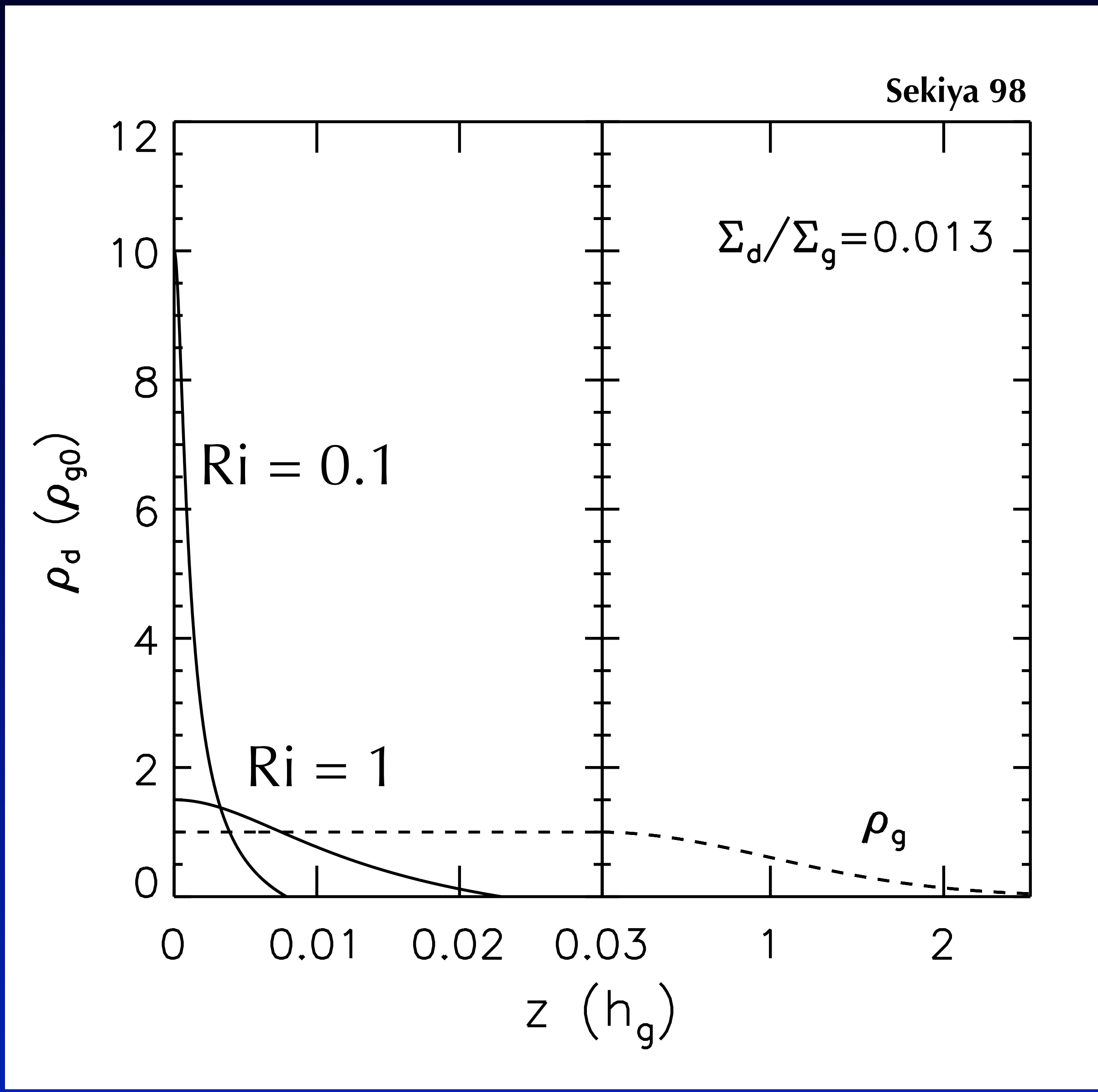
$$= \left[\frac{1}{1/(1+\mu_0)^2 + (z/z_d)^2}\right]^{1/2} - 1$$

where $z_d = \frac{Ri^{1/2}\Delta v}{\Omega}$

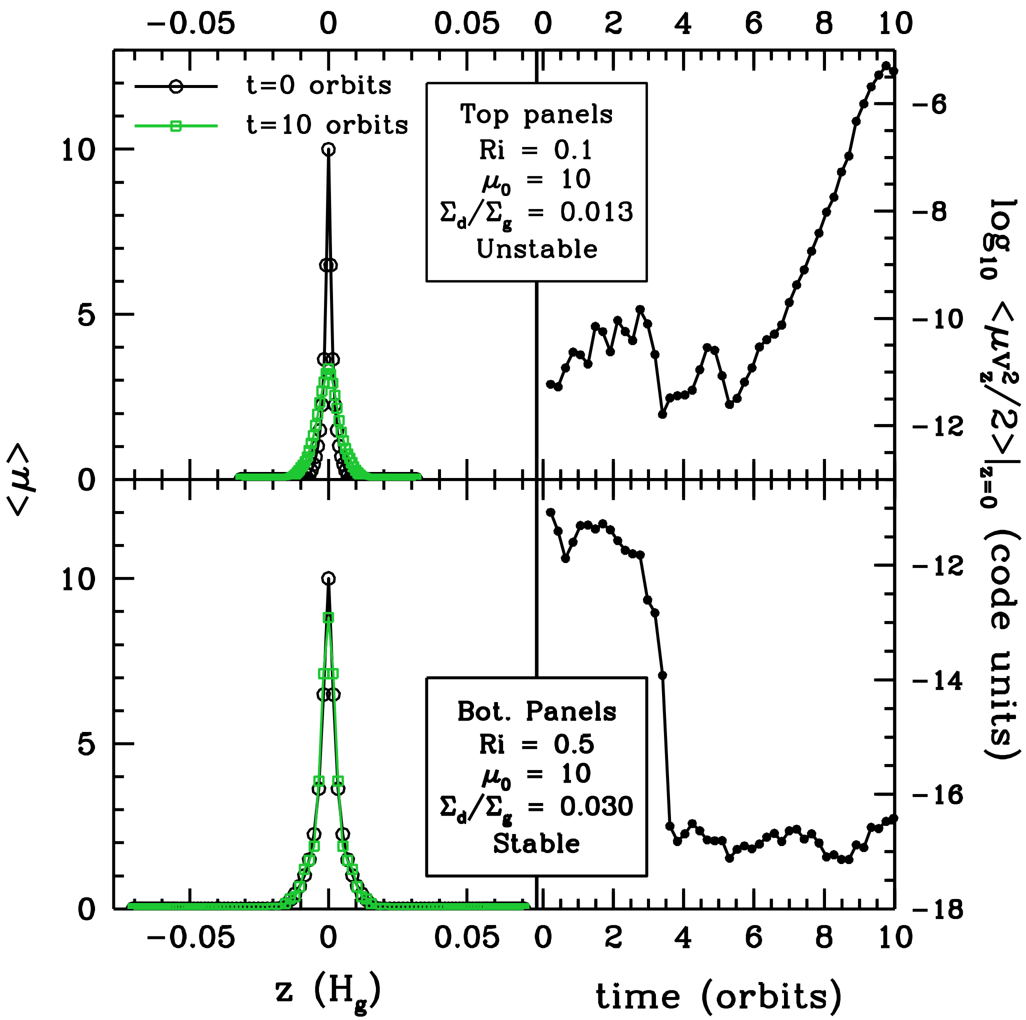
Input parameters:

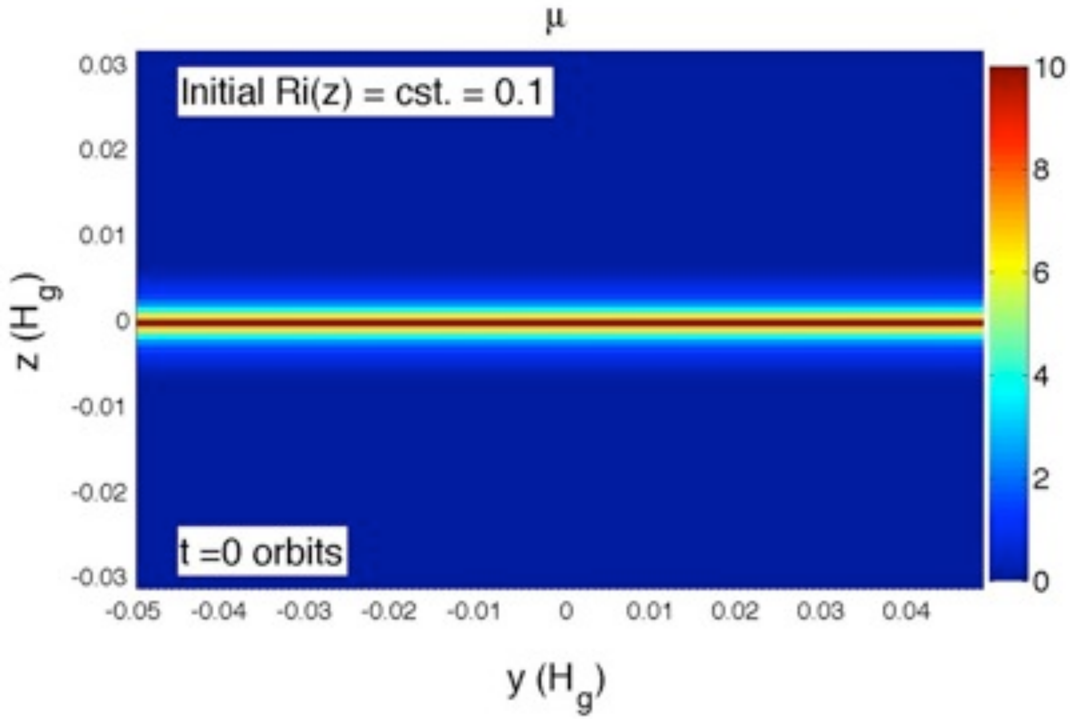
$(\mathrm{Ri}, \mu_0)$

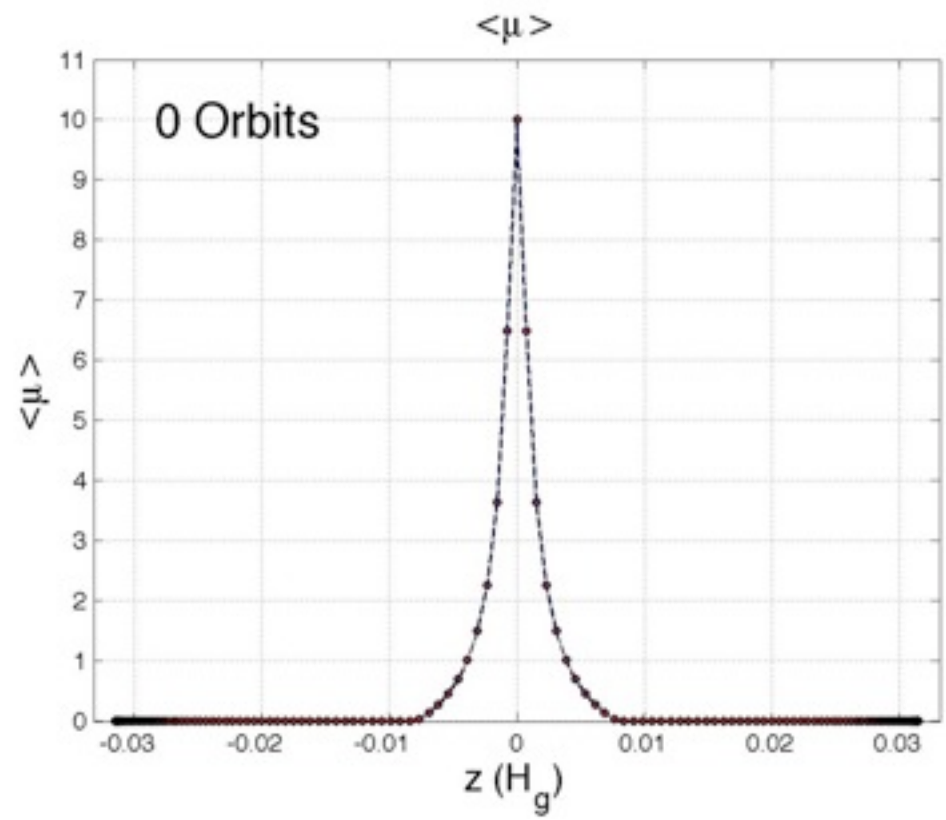
$(\mathrm{Ri}, \Sigma_d/\Sigma_g) \longleftrightarrow (\mu_0, \Sigma_d/\Sigma_g)$



# Numerical simulations of dense midplanes

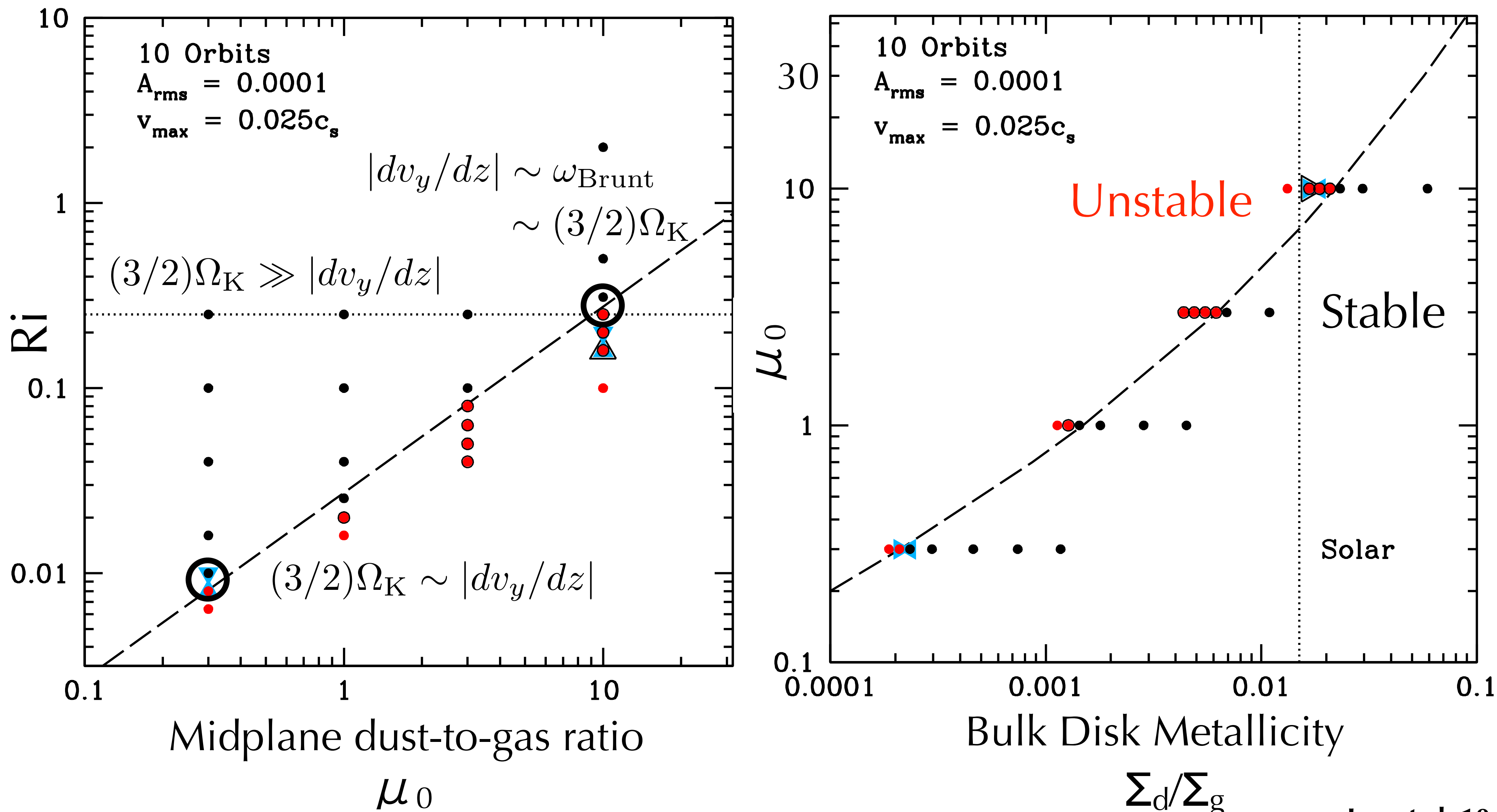


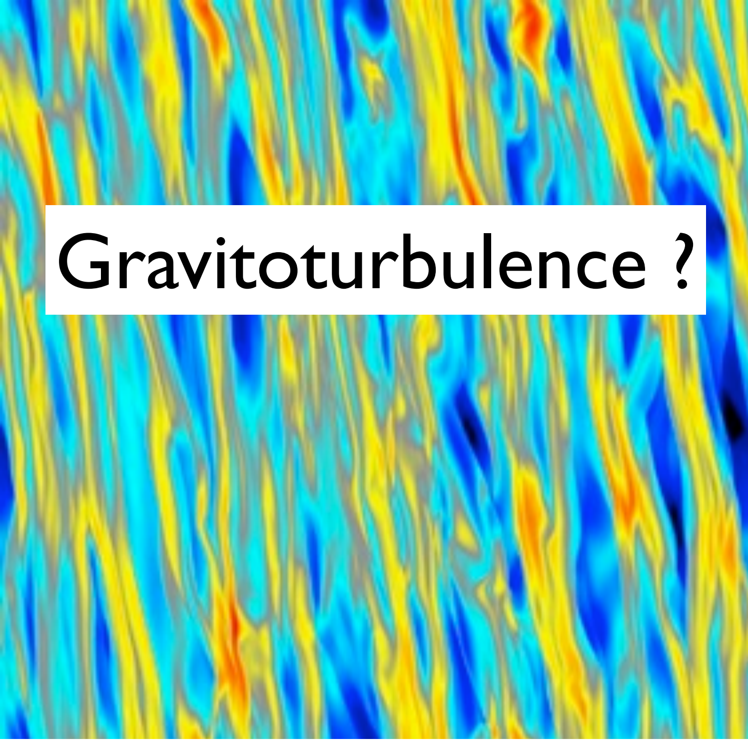

EC 08; Lee, EC, Asay-Davis, and Barranco 10a

# Gravitational Instability by Metal Enrichment

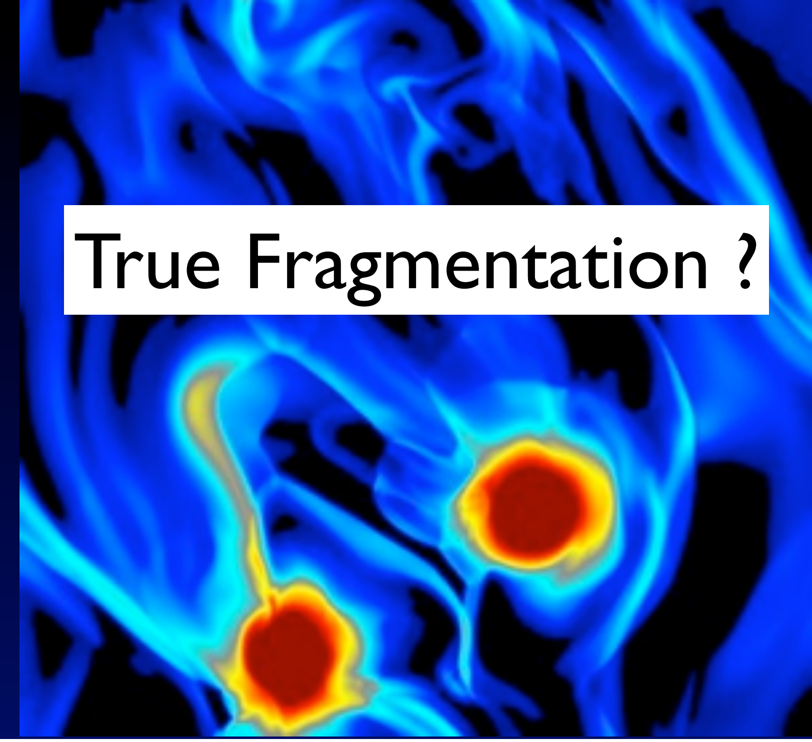


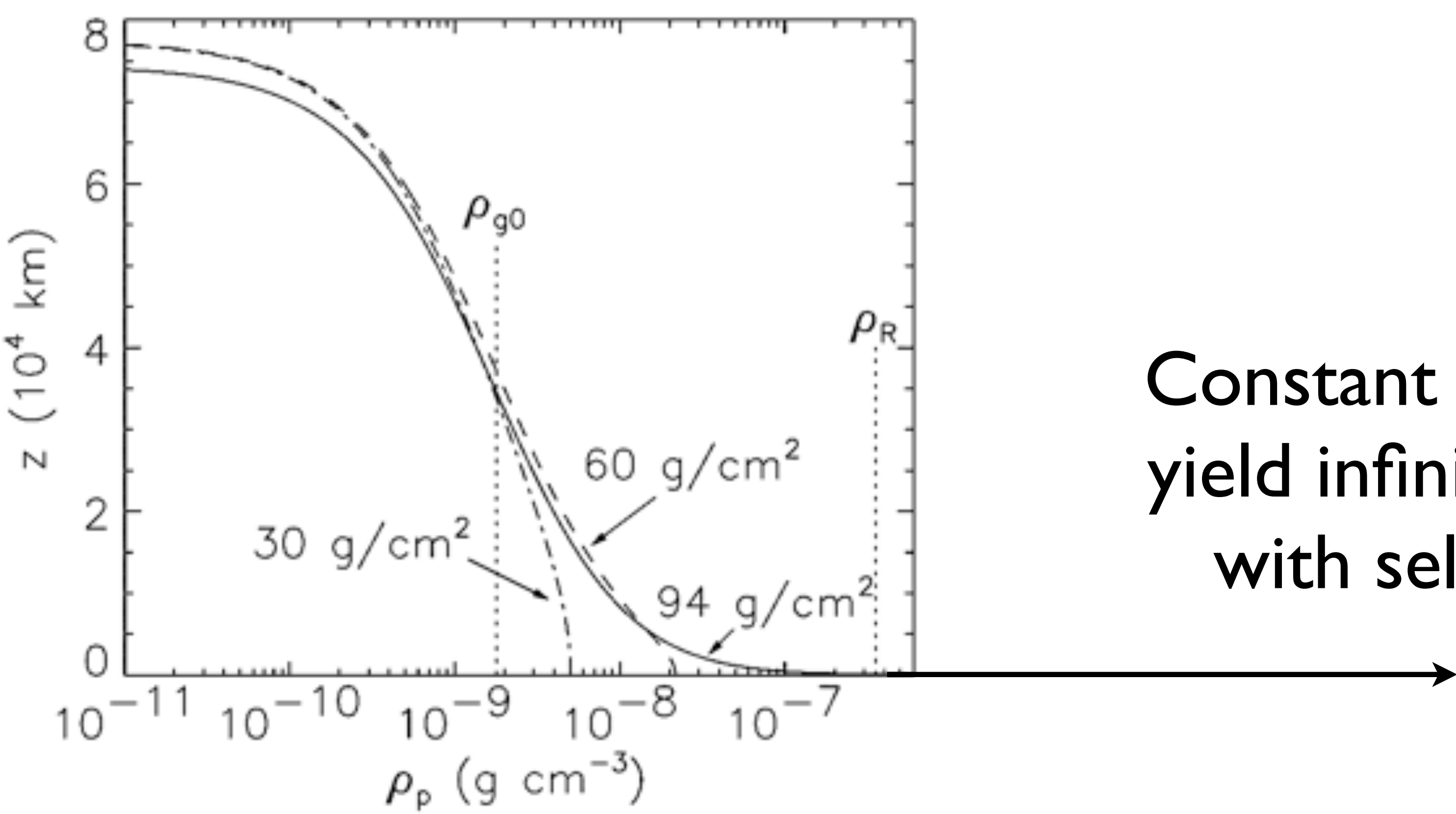

Lee et al. 10a

Toomre-like densities possible for < 4x bulk solar metallicity,
or < 4x more mass than minimum-mass solar nebula



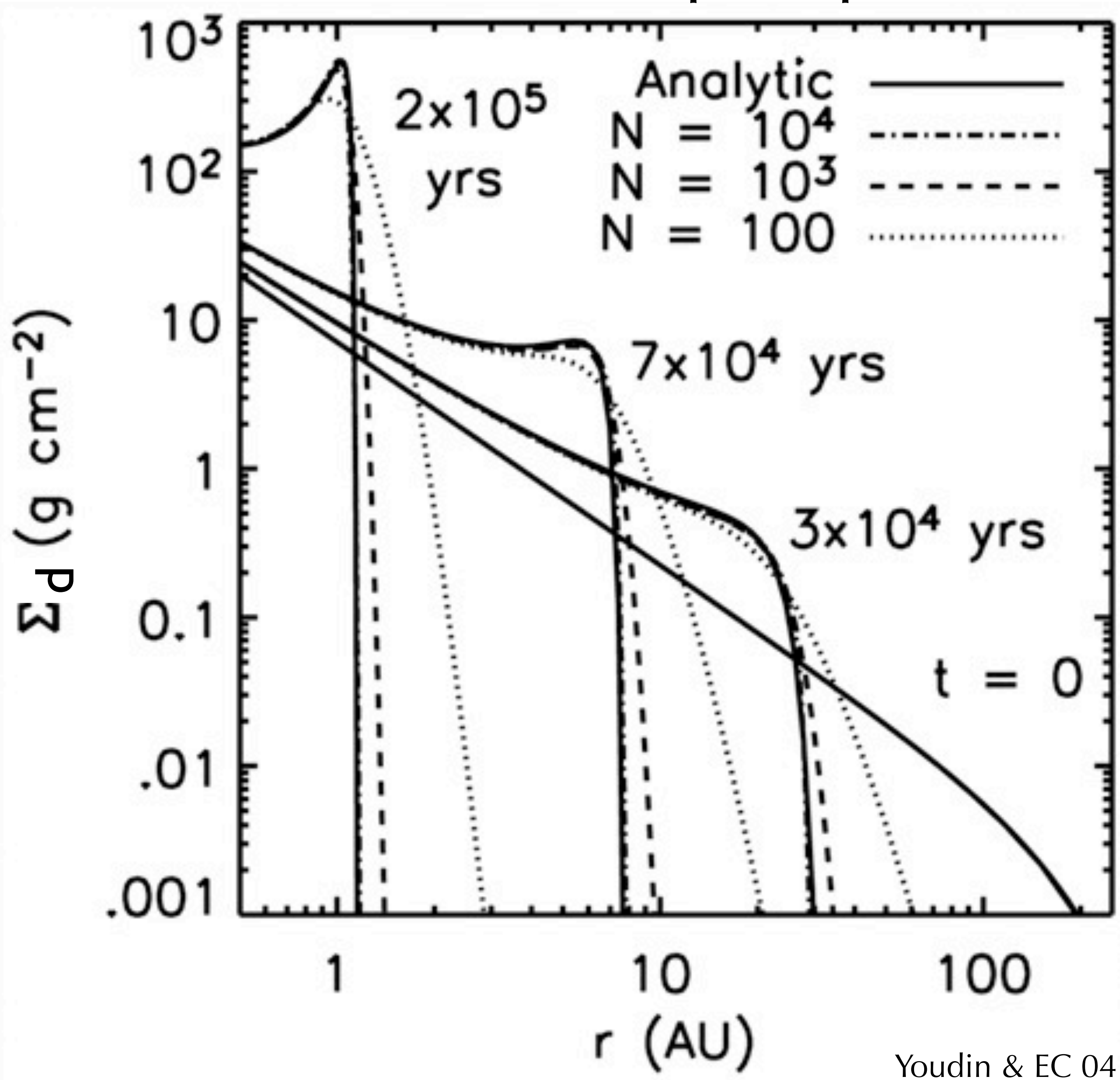

or



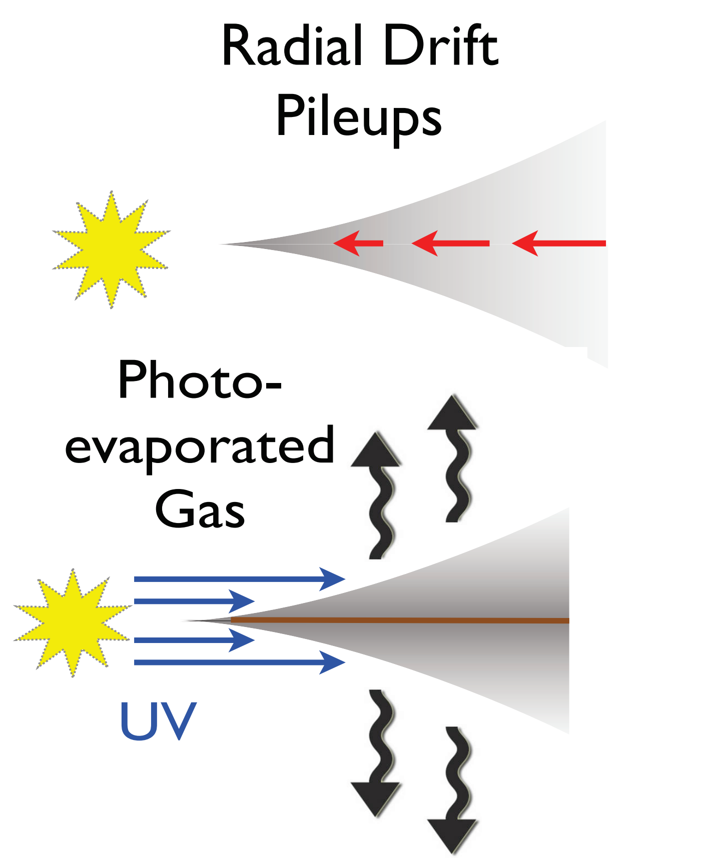




Constant Ri profiles yield infinite density with self-gravity

# *Local* enrichments of metallicity



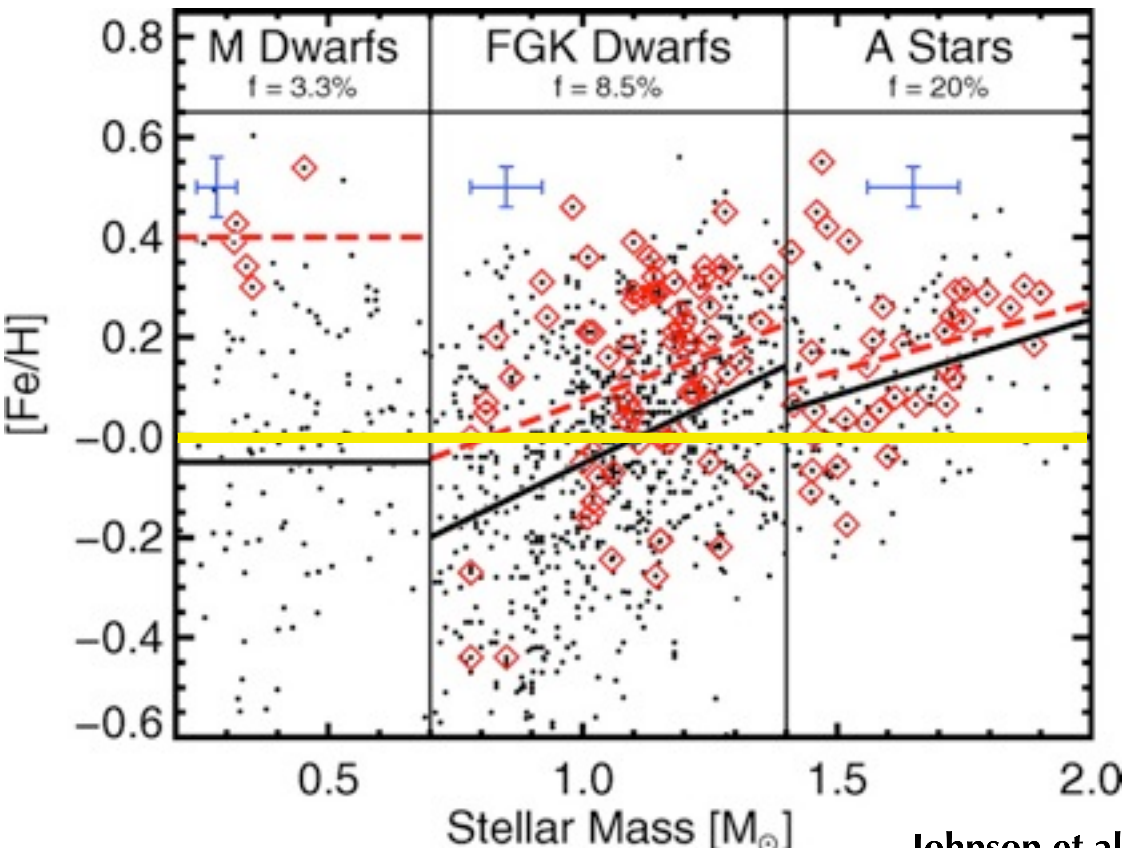

Youdin & EC 04



# Modeled metallicities

**Guillot et al. 06**

| Name | $M_{planet}$ ($M_\oplus$) | $M_Z$ [a] ($M_\oplus$) | $Z_{planet}$ ($M_Z/M_{planet}$) | $Z_{planet}/Z_\odot$ [b] | [Fe/H]$_*$ | $Z_{planet}/Z_*$ |
|---|---|---|---|---|---|---|
| HD209458 | 210 | 20 | 0.095 | 6.35 | 0.02 | 6.06 |
| OGLE-TR-56 | 394 | 120 | 0.304 | 20.3 | 0.25 | 11.418 |
| OGLE-TR-113 | 429 | 70 | 0.163 | 10.9 | 0.15 | 7.7 |
| OGLE-TR-132 | 350 | 105 | 0.3 | 20 | 0.37 | 8.531 |
| OGLE-TR-111 | 168 | 50 | 0.297 | 19.84 | 0.19 | 12.81 |
| OGLE-TR-10 | 200 | 10 | 0.05 | 3.33 | 0.28 | 1.75 |
| TrES-1 | 238 | 50 | 0.21 | 14.0 | 0.06 | 12.2 |
| HD149026 | 114 | 80 | 0.70 | 46.78 | 0.36 | 20.42 |
| HD189733 | 365 | 30 | 0.082 | 5.479 | -0.03 | 5.87 |
| Jupiter | 318 | 10–42 | 0.03–0.13 | 2.0–8.8 | 0 | 2.0–8.8 |
| Saturn | 95.2 | 15–30 | 0.16–0.32 | 11–21 | 0 | 11–21 |




**Johnson et al. 10**

Toomre density requires:

Super-solar bulk metallicities

(< 4 x; $\Sigma_d/\Sigma_g = 0.05$)

and/or

Disk masses >

minimum-mass nebula (< 4 x)

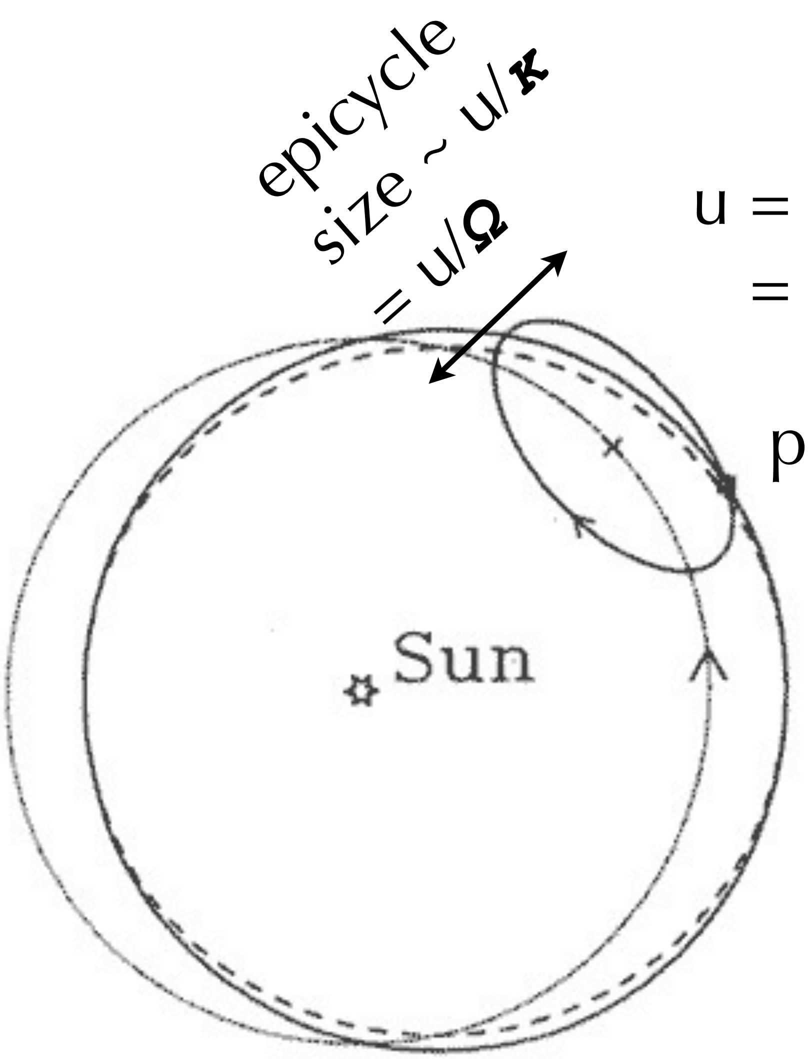
Elliptical motion = Guiding center + Epicycle

$\Omega$ = guiding center (azimuthal) frequency

$\kappa$ = epicyclic (radial) frequency

"Kepler degeneracy": $\kappa = \Omega$

u = epicyclic velocity
= small body random velocity



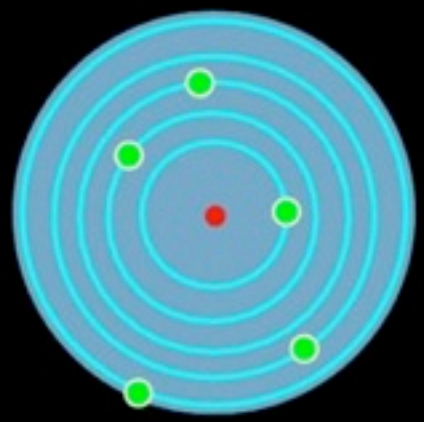

particle

**Figure 3-6.** An elliptical Kepler orbit (dashed curve) is well approximated by the superposition of retrograde motion at angular frequency $\kappa$ around a small ellipse with axis ratio $\frac{1}{2}$, and prograde motion of the ellipse's center at angular frequency $\Omega$ around a circle (dotted curve).

# Massive disks of small bodies

(Goldreich, Lithwick & Sari 04, ARAA)

big bodies: $R, \Sigma, v \ll u$



small bodies: $s, \sigma, u$

without gravitational focussing:

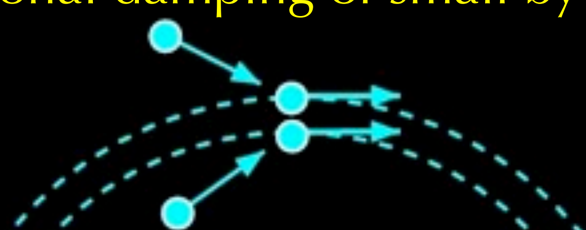
$$t_{\rm acc} \sim \frac{M}{\dot{M}} \sim \frac{\rho R}{\sigma \Omega} \sim 10^{12}\,\mathrm{yr}$$

with gravitational focussing: $t_{\rm acc} \sim \frac{\rho R}{\sigma \Omega}\left(\frac{u}{v_{\rm esc}}\right)^2 \sim 10^7\,\mathrm{yr}$

$$\frac{\rho R}{\Sigma \Omega}\left(\frac{u}{v_{\rm esc}}\right)^4 \sim \frac{\rho s}{\sigma \Omega}$$

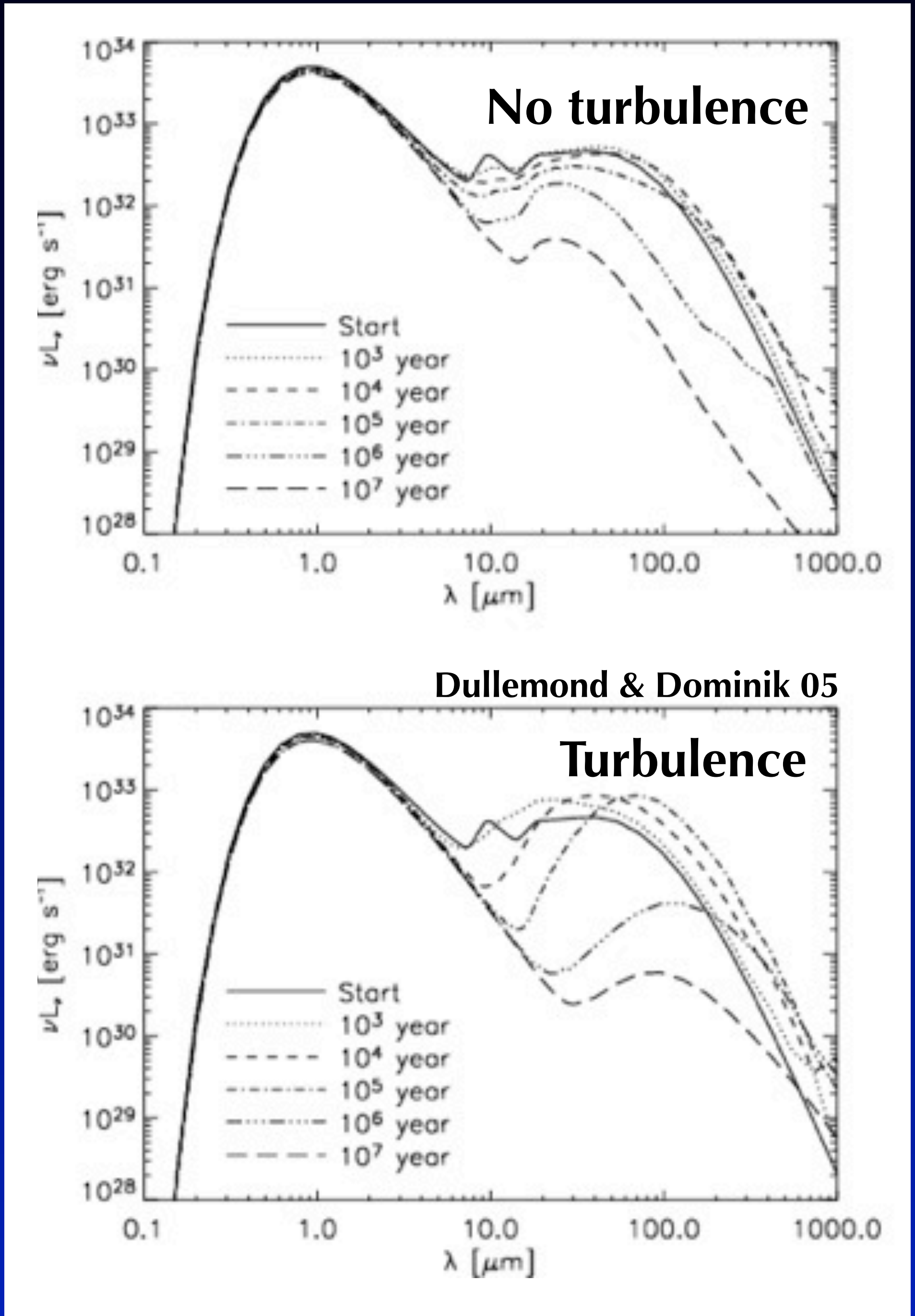
viscous stirring of small by big

collisional damping of small by small



# Extra Slides

# Grain growth: Too fast




Sticks too well

Problem persists even if

- grains are fractal
- monomers are nonspherical

Proposed solution:

Replenishment of micron-sized grains (near-IR opacity) by fragmentation

For small particles well coupled to gas ( $\tau_s << 1$):

1. Is the Richardson criterion a good predictor of stability?

    - Doesn't formally apply because flow is 3D and rotational
    - Brunt vs. vertical shear vs. Coriolis vs. Kepler shear
    - Coriolis is destabilizing
    - Kepler shear is stabilizing

2. How does maximum dust density $\rho_d$ depend on bulk (height-integrated) metallicity $\Sigma_d/\Sigma_g$?

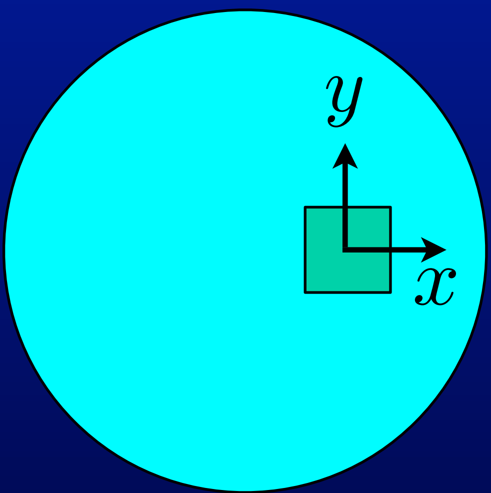
    - Disk metallicity may be supersolar
    - Host stars of extrasolar planets tend to be metal-rich
    - Planets themselves are metal-rich

Ishitsu & Sekiya 03; Gomez & Ostriker 05; Johnson et al. 10; Guillot et al. 06

# Well-coupled gas and dust in a shearing box

$$\frac{\partial v_x}{\partial t} + \mathbf{v} \cdot \nabla v_x = \frac{-1}{\rho + \rho_d} \frac{\partial P}{\partial x} + 2\Omega_0 v_y + 2q\Omega_0^2 x \quad - \frac{\left(\frac{\partial P}{\partial x}\right)_0}{\rho + \rho_d}$$

$$\frac{\partial v_y}{\partial t} + \mathbf{v} \cdot \nabla v_y = \frac{-1}{\rho + \rho_d} \frac{\partial P}{\partial y} - 2\Omega_0 v_x$$



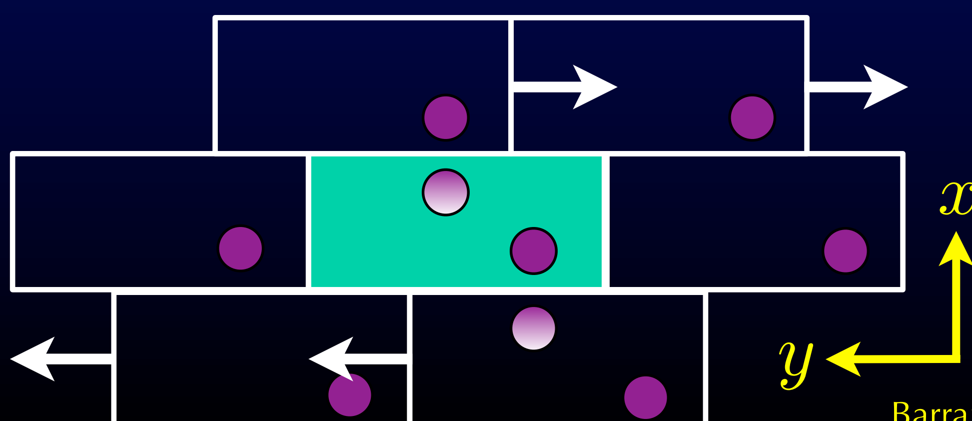

$$\frac{\partial v_z}{\partial t} + \mathbf{v} \cdot \nabla v_z = \frac{-1}{\rho + \rho_d} \frac{\partial P}{\partial z} - \Omega_0^2 z$$

anelastic

$$\cancel{\frac{\partial \rho}{\partial t}} + \nabla \cdot (\rho_0 \mathbf{v}) = 0 \qquad \frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{v}) = 0$$

$$\frac{\partial \varepsilon}{\partial t} + \mathbf{v} \cdot \nabla \varepsilon = -(P + \varepsilon)\nabla \cdot \mathbf{v}$$

$$P = (\gamma - 1)\varepsilon$$

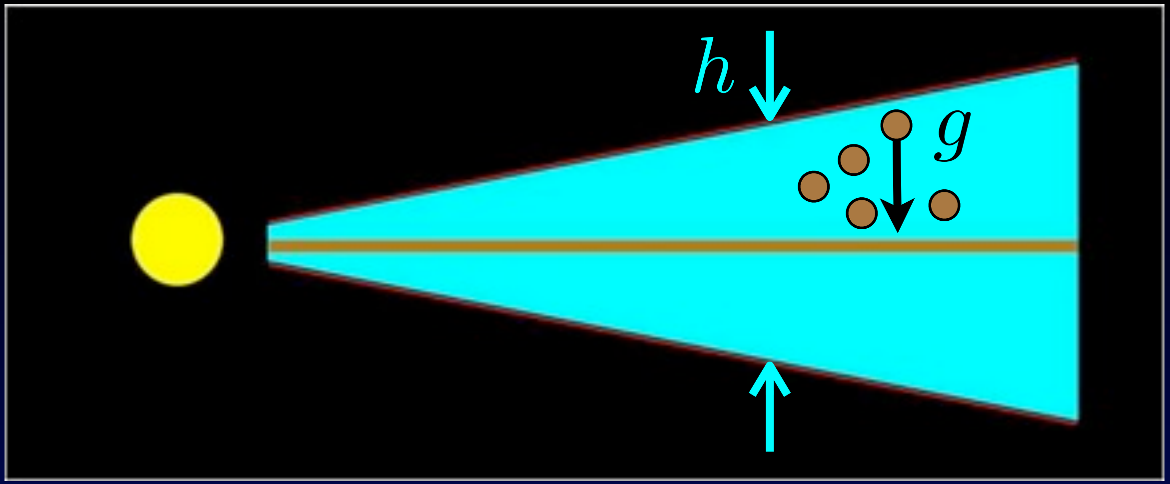



Barranco 09

Code limitations (so far):

1. No self-gravity
   - Use Toomre density as guide for onset of gravitational instability
2. Dust and gas are perfectly coupled
   - Restricted to studying stability of given initial conditions

# Grain growth




$$mg \sim F_{\mathrm{drag}}(v)$$

$$\mu s^3 \Omega^2 h \sim \rho_g s^2 c_s v$$

$\longrightarrow$ Terminal $v \sim \dfrac{\mu}{\rho_g}\Omega s$ (bigger is faster)

Accretion $\dfrac{d}{dt}(\mu s^3) \sim \rho_d v s^2 \longrightarrow \dot{s} \sim \dfrac{\rho_d}{\mu} v$ (faster is bigger)

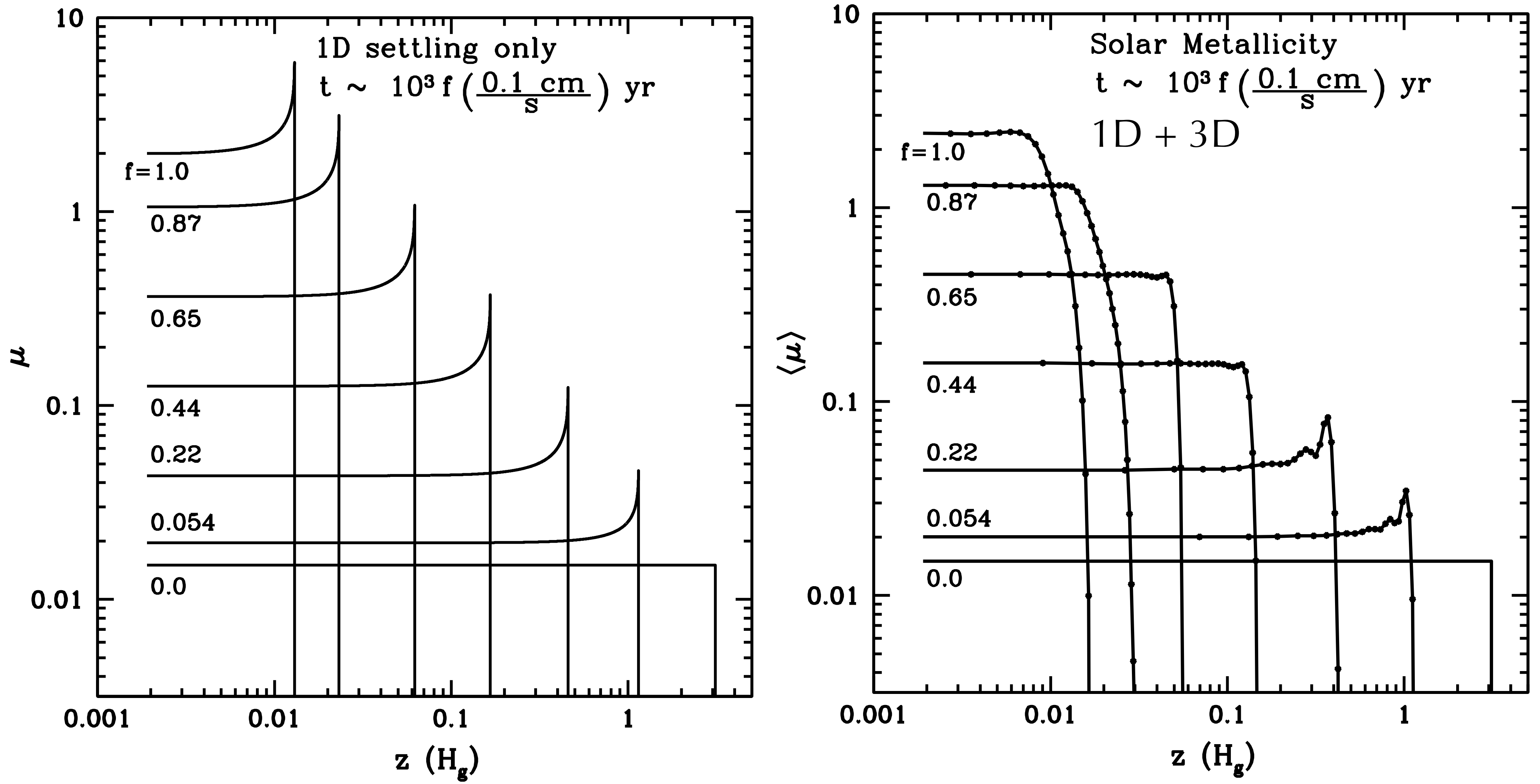
$\longrightarrow$ Exponential growth $s \sim s_0 \exp(\rho_d \Omega t/\rho_g)$ (fastest growth in inner disk)

Since $t \sim h/v \longrightarrow s \sim s_0 \exp(\Sigma_d/\mu s)$

$$\left.\begin{array}{r} s_0 \sim 1\,\mu\mathrm{m} \\ \mu \sim 1\,\mathrm{g\,cm^{-3}} \\ \Sigma_d \sim 10\,\mathrm{g\,cm^{-2}} \end{array}\right\} \quad \begin{array}{l} s \sim 1\,\mathrm{cm} \\ t \sim 100\,\mathrm{yr} \\ v \sim 1\,\mathrm{m/s} \end{array}$$

Safronov 1969

# Relaxing constant Ri: Settling from arbitrary initial conditions



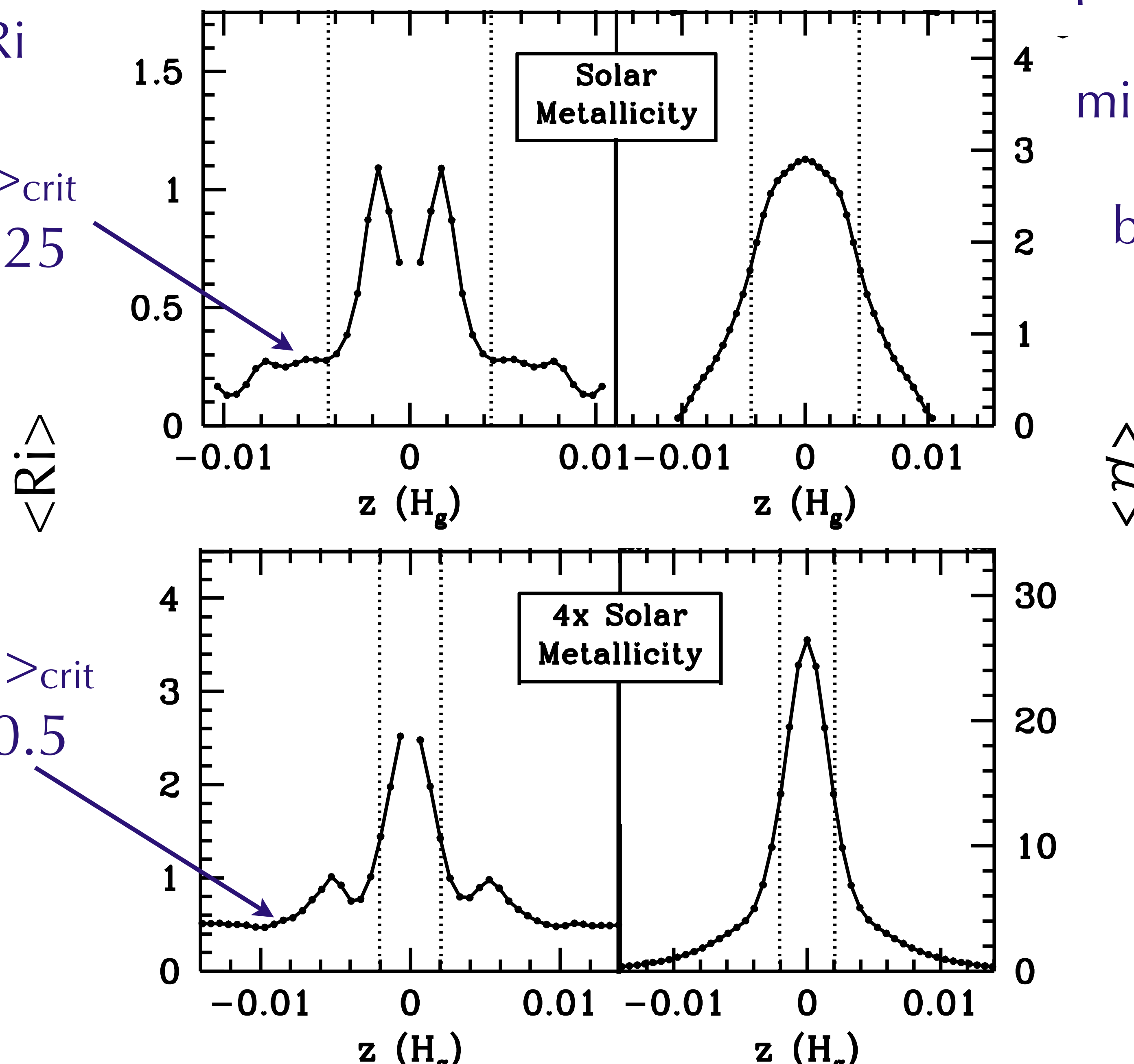

Lee et al. 10b

Finding the marginally stable state

# Marginally stable states

Evidence for constant Ri

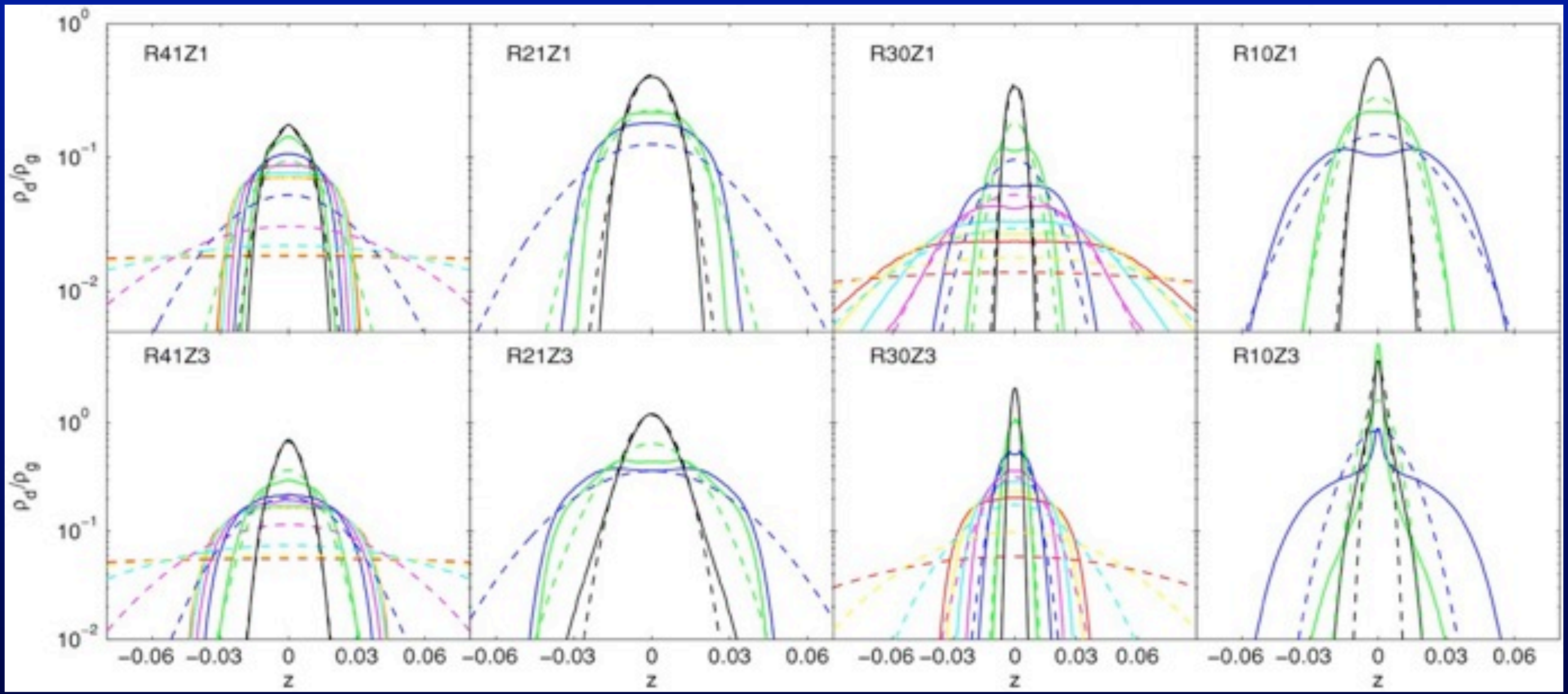
Superlinear relation between midplane density and bulk metallicity



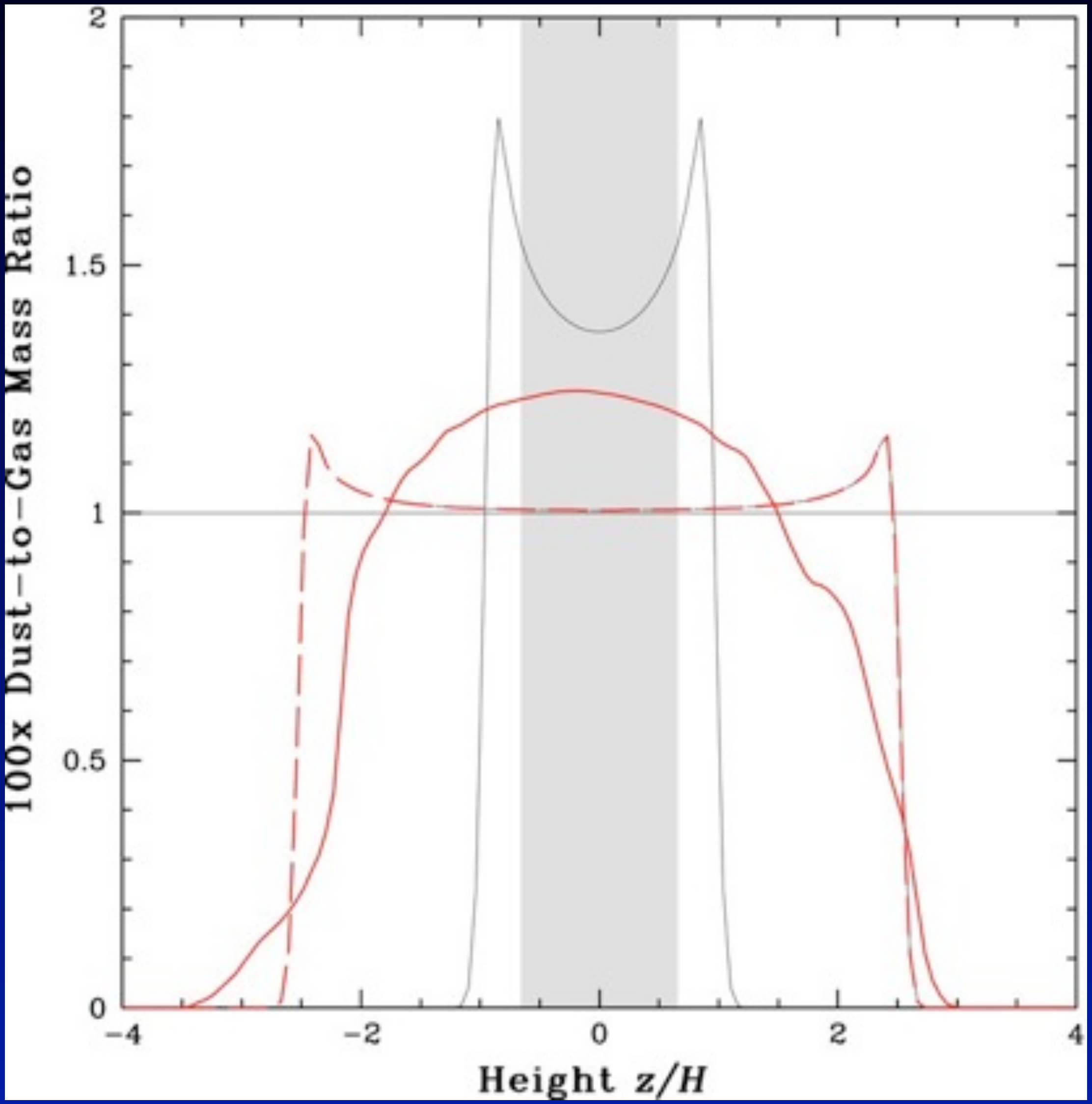

$\langle Ri \rangle_{crit} \approx 0.25$

$\langle Ri \rangle_{crit} \approx 0.5$

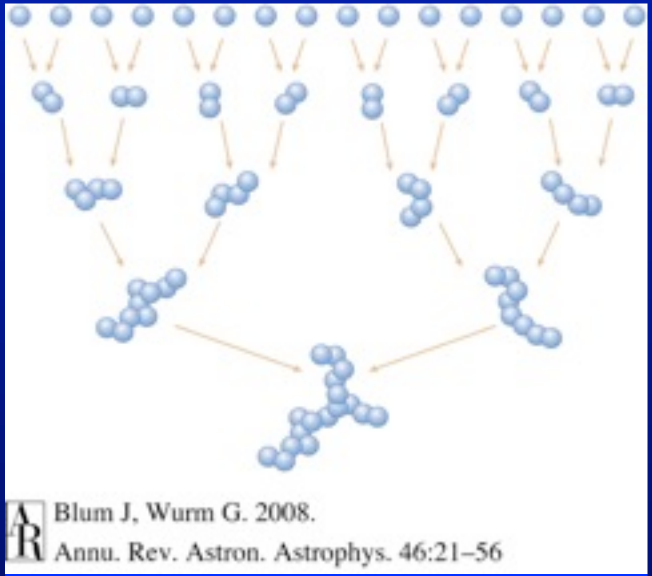
$\langle \mu_0 \rangle \propto (\Sigma_d/\Sigma_g)^{>1}$

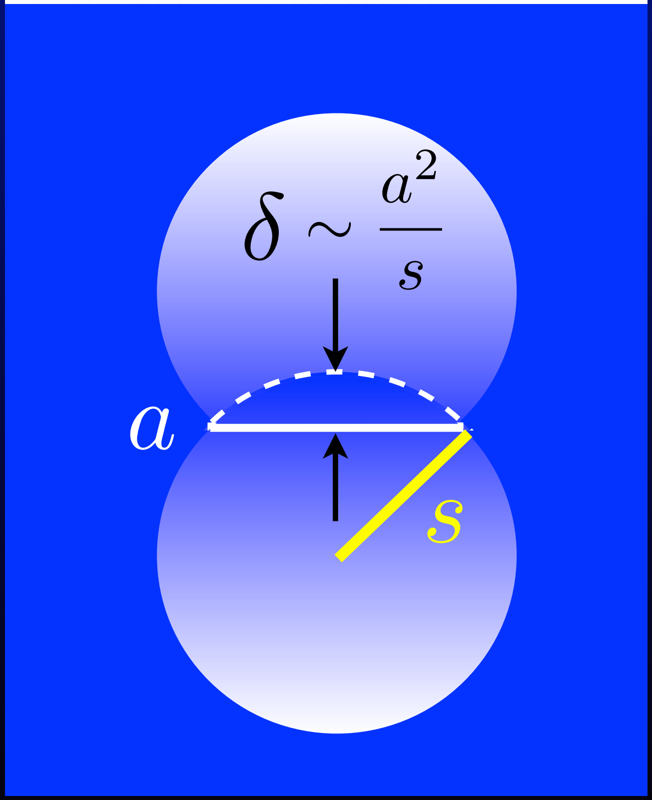


R41Z1
R21Z1
R30Z1
R10Z1
R41Z3
R21Z3
R30Z3
R10Z3
$\rho_d/\rho_g$
$10^0$
$10^{-1}$
$10^{-2}$
−0.06 −0.03 0 0.03 0.06
z


100x Dust-to-Gas Mass Ratio
Height z/H
2
1.5
1
0.5
0
−4
−2
0
2
4

# Grain growth




$$\boxed{v_{\rm crit} \sim 1\,{\rm m/s} \text{ for } s \sim 1\,\mu{\rm m}}$$

Repulsion (elastic modulus E)

$$\text{Stress}\, \sigma \sim E\nabla\xi \sim E\frac{\delta}{a}$$

$$\sigma \sim \frac{mv}{(\delta/v) \times a^2}$$

$$\text{Repulsive force}\, F_R \sim \sigma a^2 \sim \mu^{3/5} E^{2/5} s^2 v^{6/5}$$

$$\text{Repulsive energy}\, U_R \sim F_R \delta$$

Adhesion (surface tension $\gamma$ )

$$\text{Binding energy}\, U_B \sim \gamma a^2$$

$$U_R = U_B \longrightarrow v_{\rm crit} \sim 4\frac{\gamma^{5/6}}{E^{1/3}\mu^{1/2}s^{5/6}}$$

Hertz 1882, Chokshi et al. 93

# Rotational Effects

Ri

| Coriolis | Kepler radial shear | Brunt oscillation | Vertical shear |
|---|---|---|---|
| $2\Omega$ | $\frac{3}{2}\Omega$ | $< \Omega$ | $< \Omega$ |
| destabilizing<br>Cabot 84<br>Gomez & Ostriker 05 | stabilizing<br>Ishitsu & Sekiya 03<br>EC 08<br>Barranco 09 | stabilizing | destabilizing |