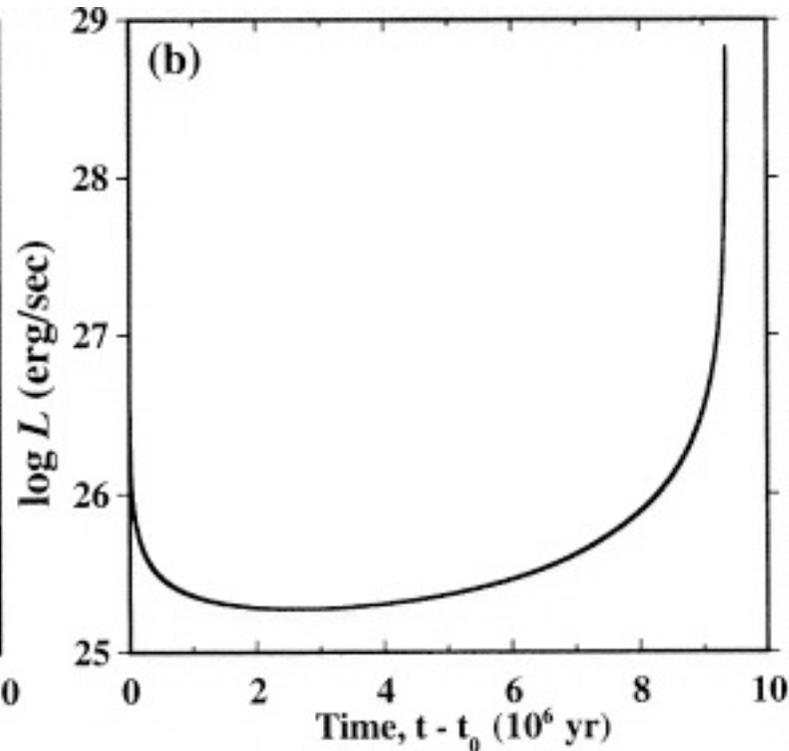
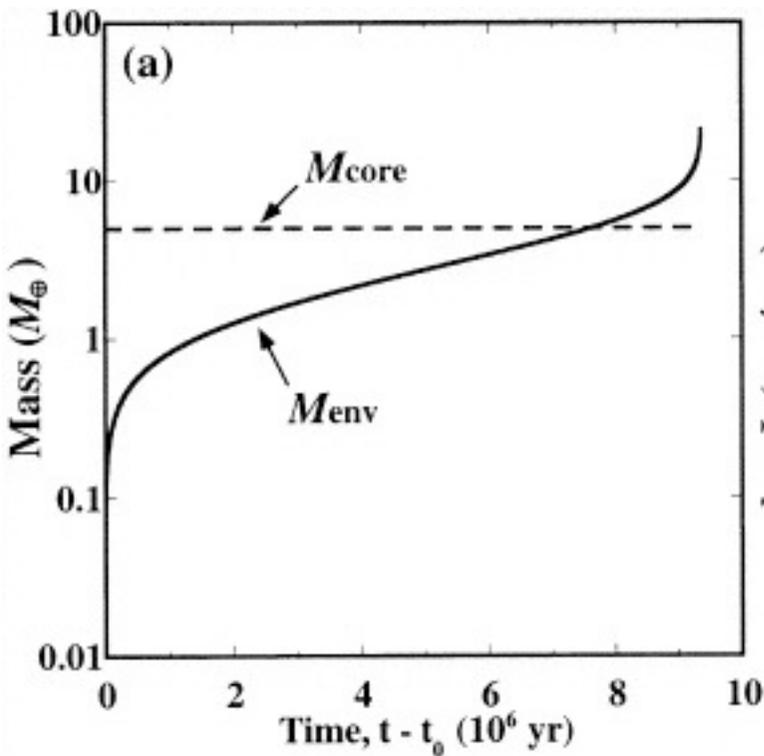
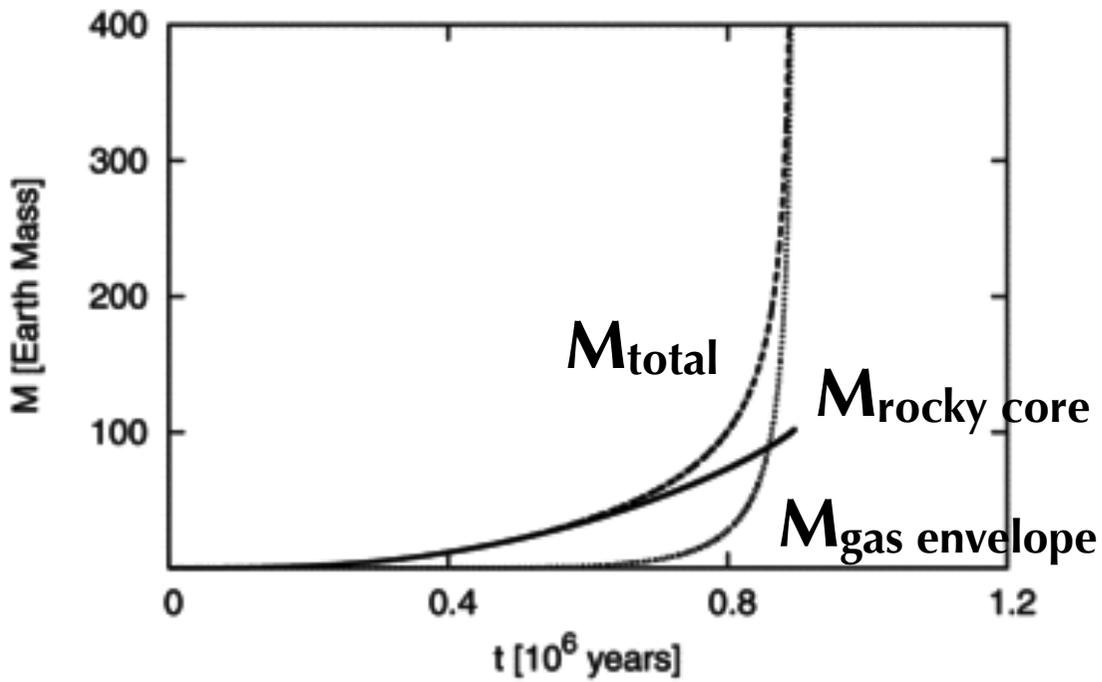
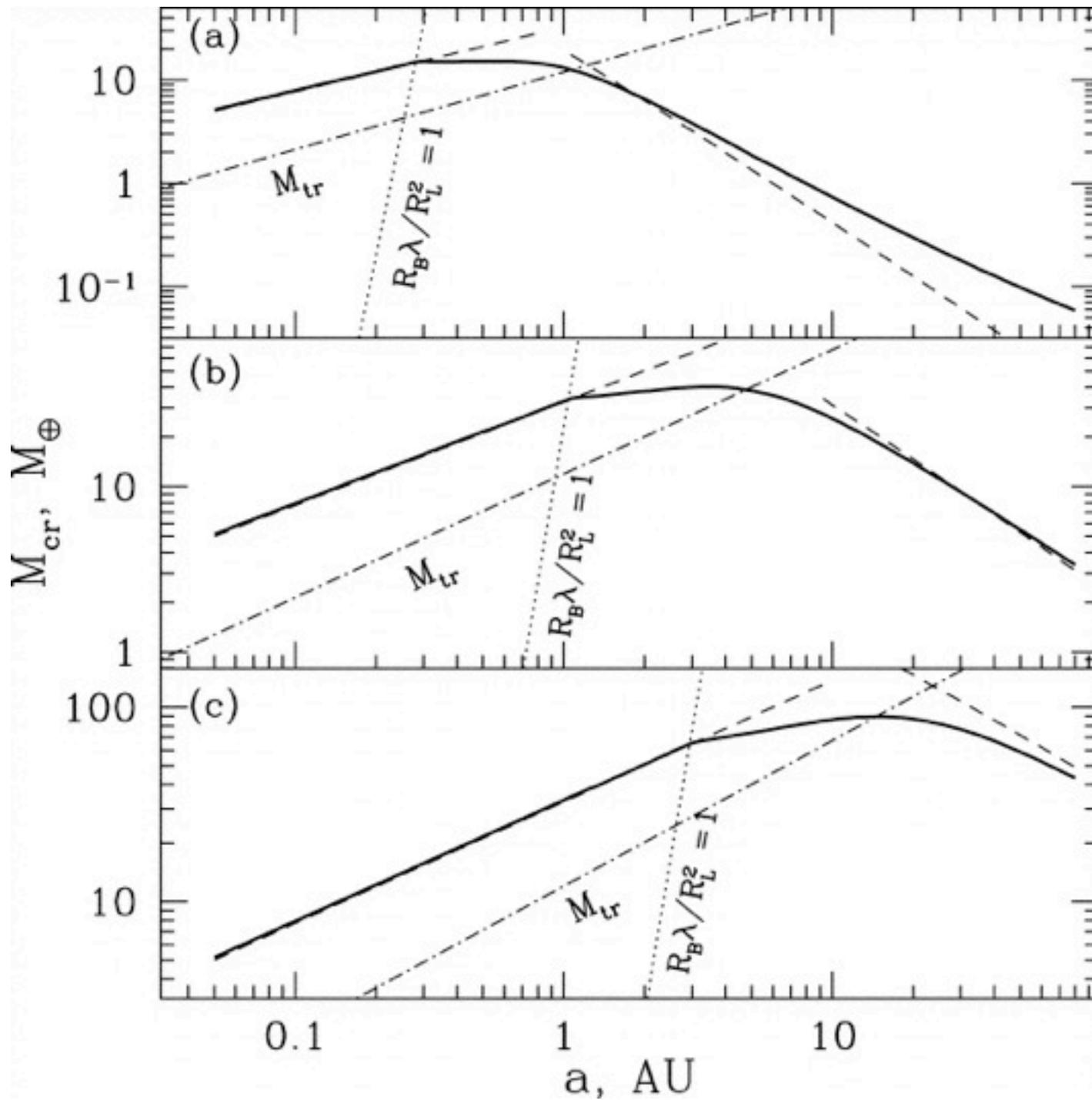


Giant planet formation

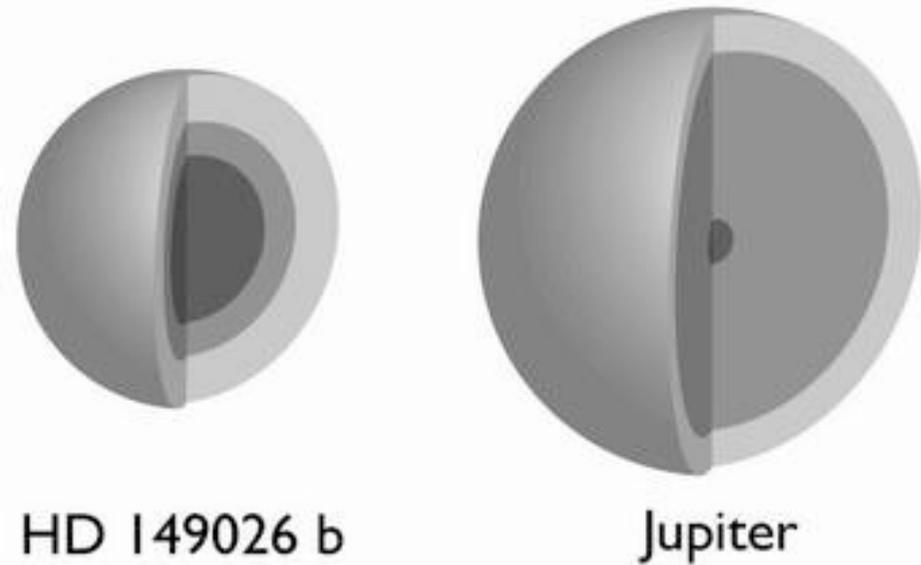
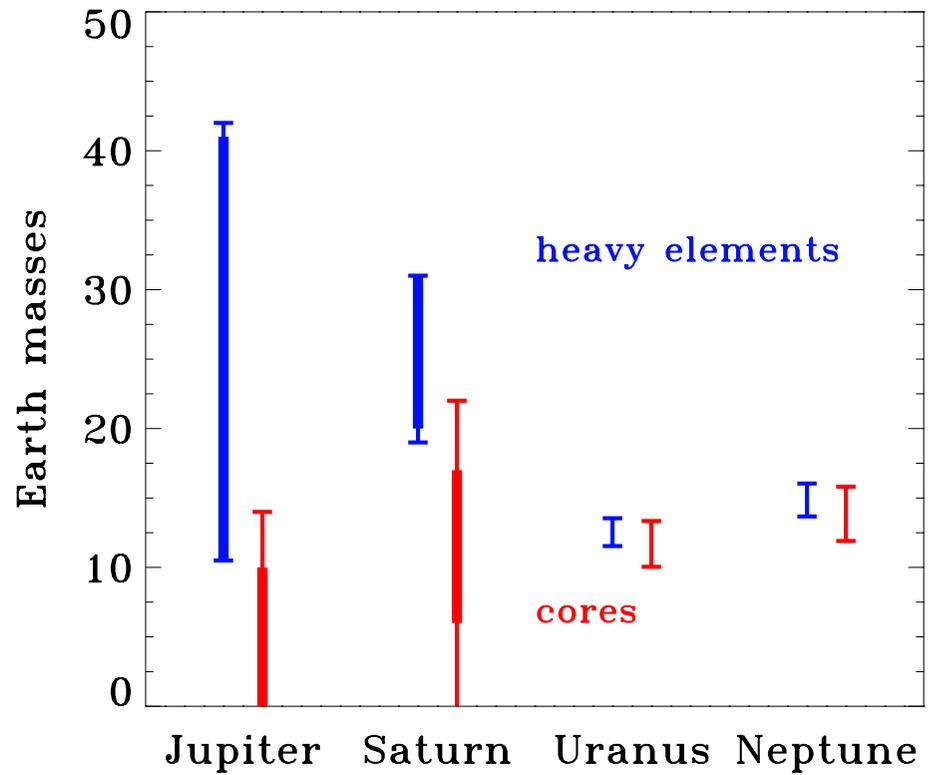
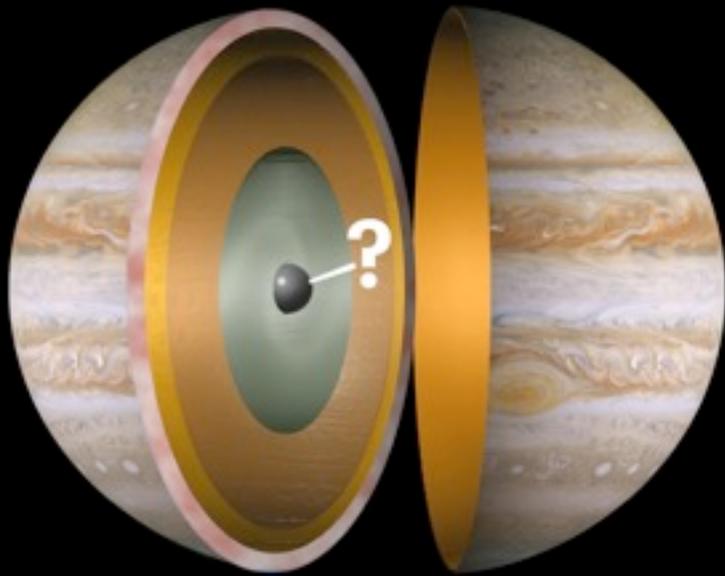
Core accretion
= core nucleation
= core instability
= "bottom-up"

Runaway gas accretion when
 $M_{\text{envelope}} \sim M_{\text{core}}$





Critical core masses for various accretion rates



Core or No Core?

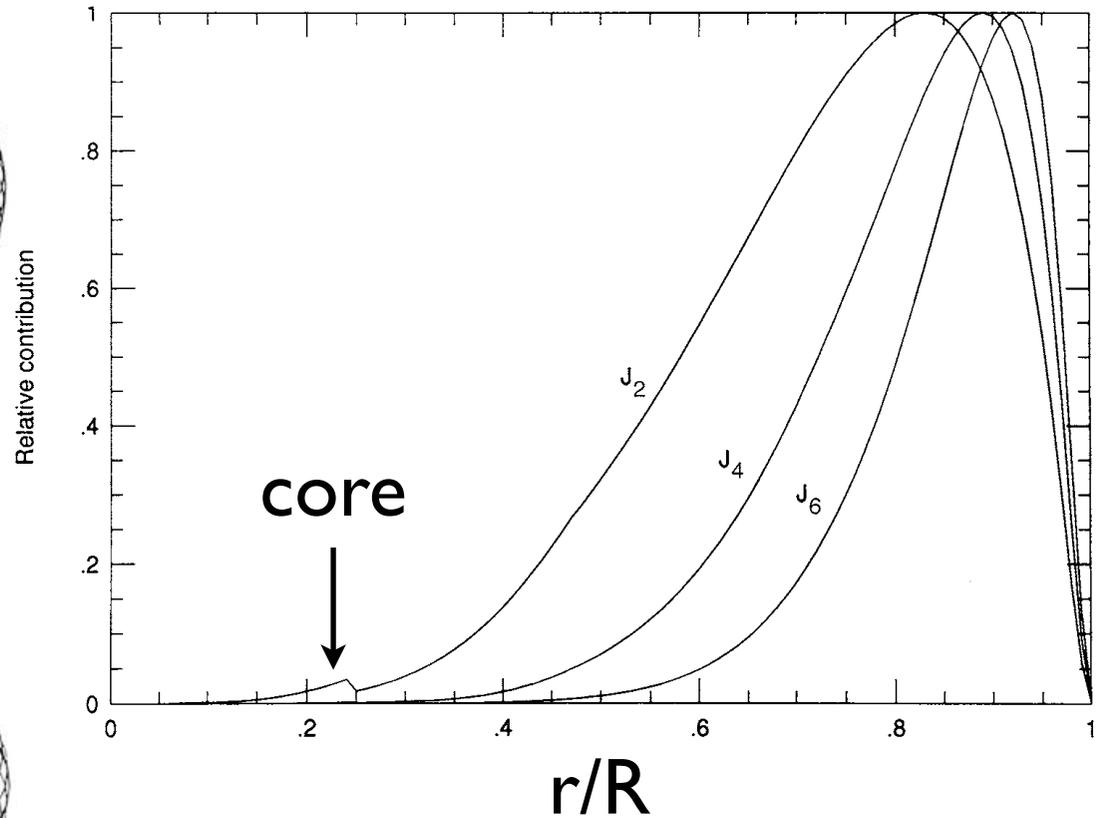
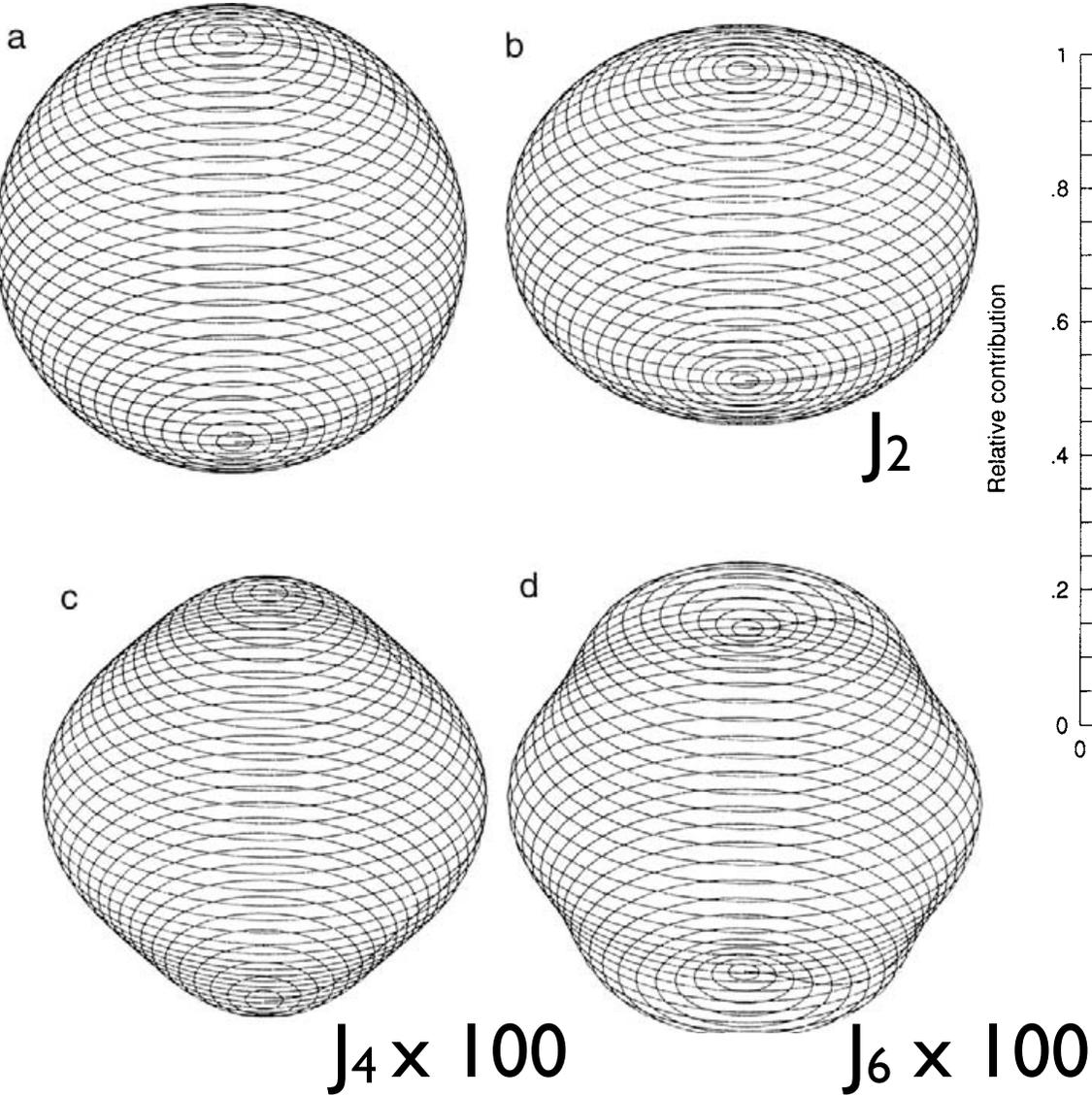
$$M, R, J_2, J_4, J_6, \dots$$

$$+ P(\rho)$$

$$+ \text{hydrostatic equilibrium (w/rotation)}$$

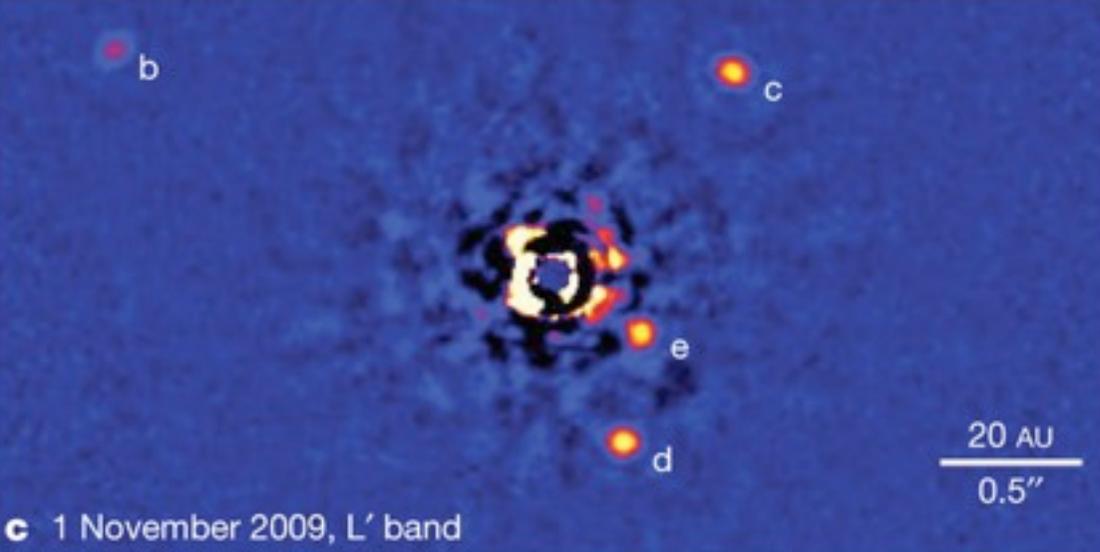
$$\Rightarrow \rho(r)$$

- hydrogen and helium gas
- liquid metallic hydrogen
- heavy element core

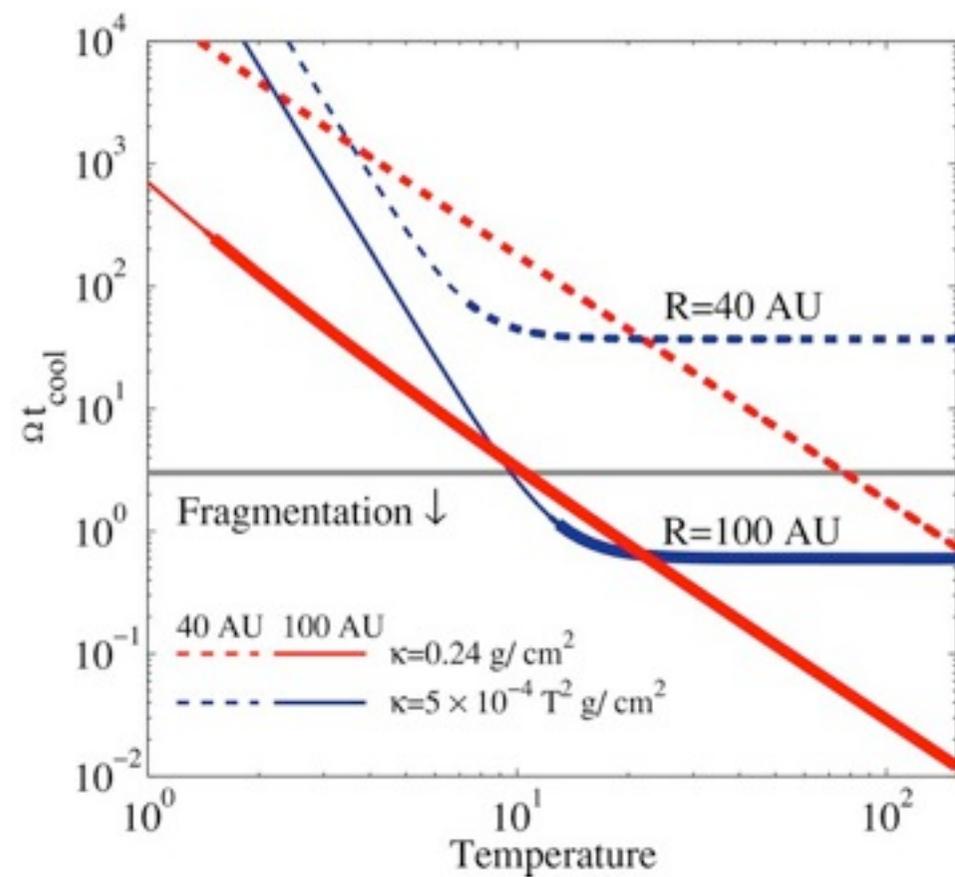


Relative contribution to
gravitational harmonics
in Saturn

$$\Phi = -\frac{GM}{r} \left(1 - \sum_{n=1}^{\infty} \left(\frac{a}{r} \right)^{2n} J_{2n} P_{2n}(\cos \theta) \right)$$



Giant planet formation by
gravitational fragmentation
= gravitational instability
= “top-down”



Kratter et al. 10

Requirements: $Q \sim 1$ and $t_{\text{cool}} < \Omega^{-1}$

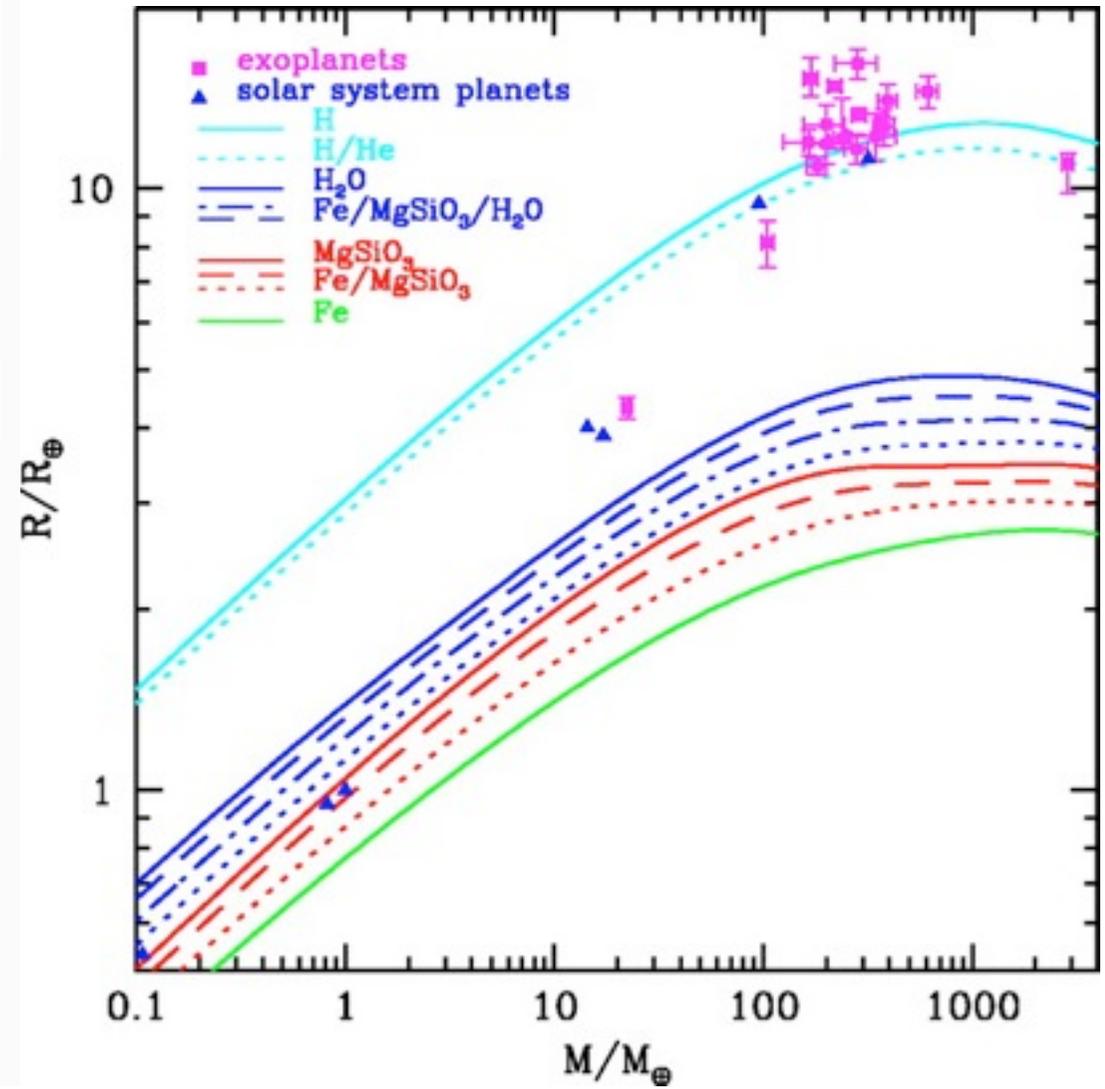
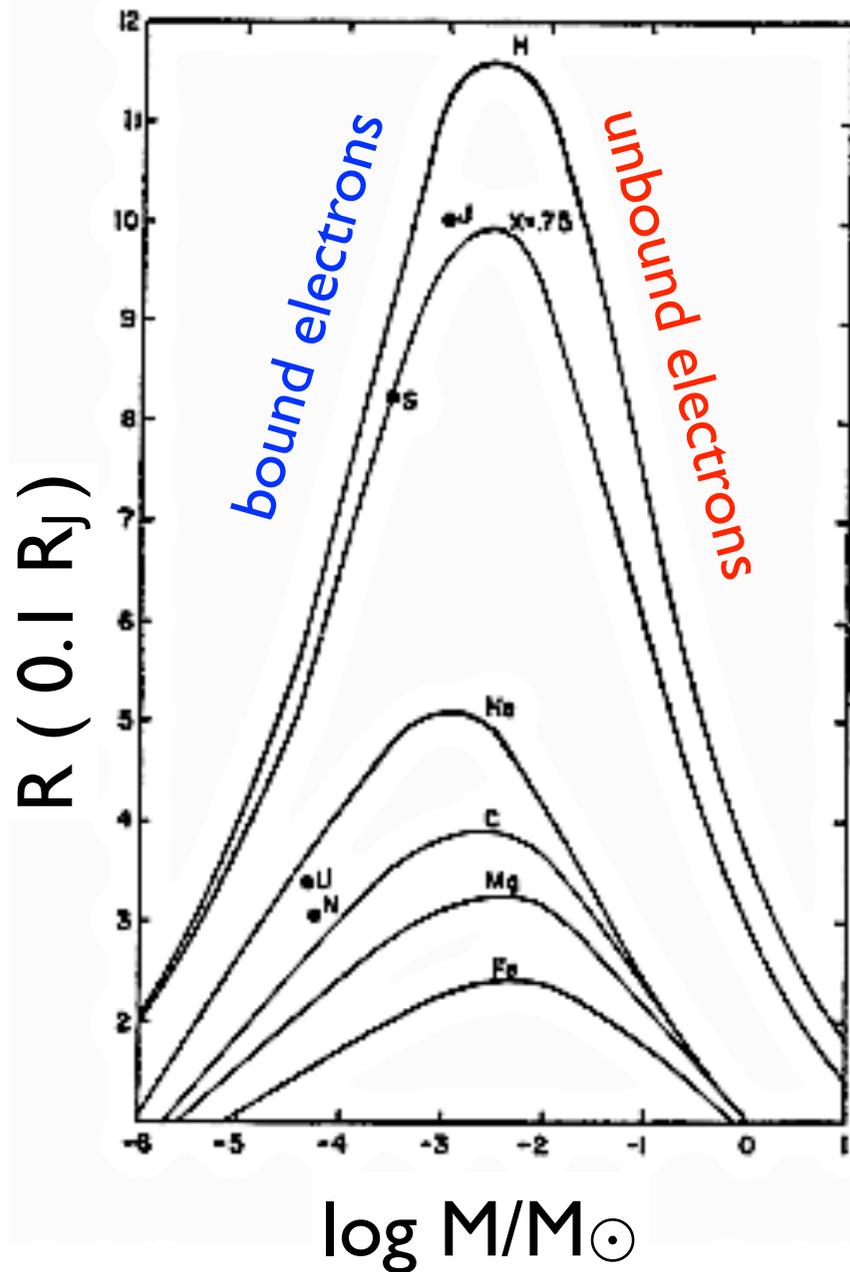
Could be met at large distance $> 70 \text{ AU}$

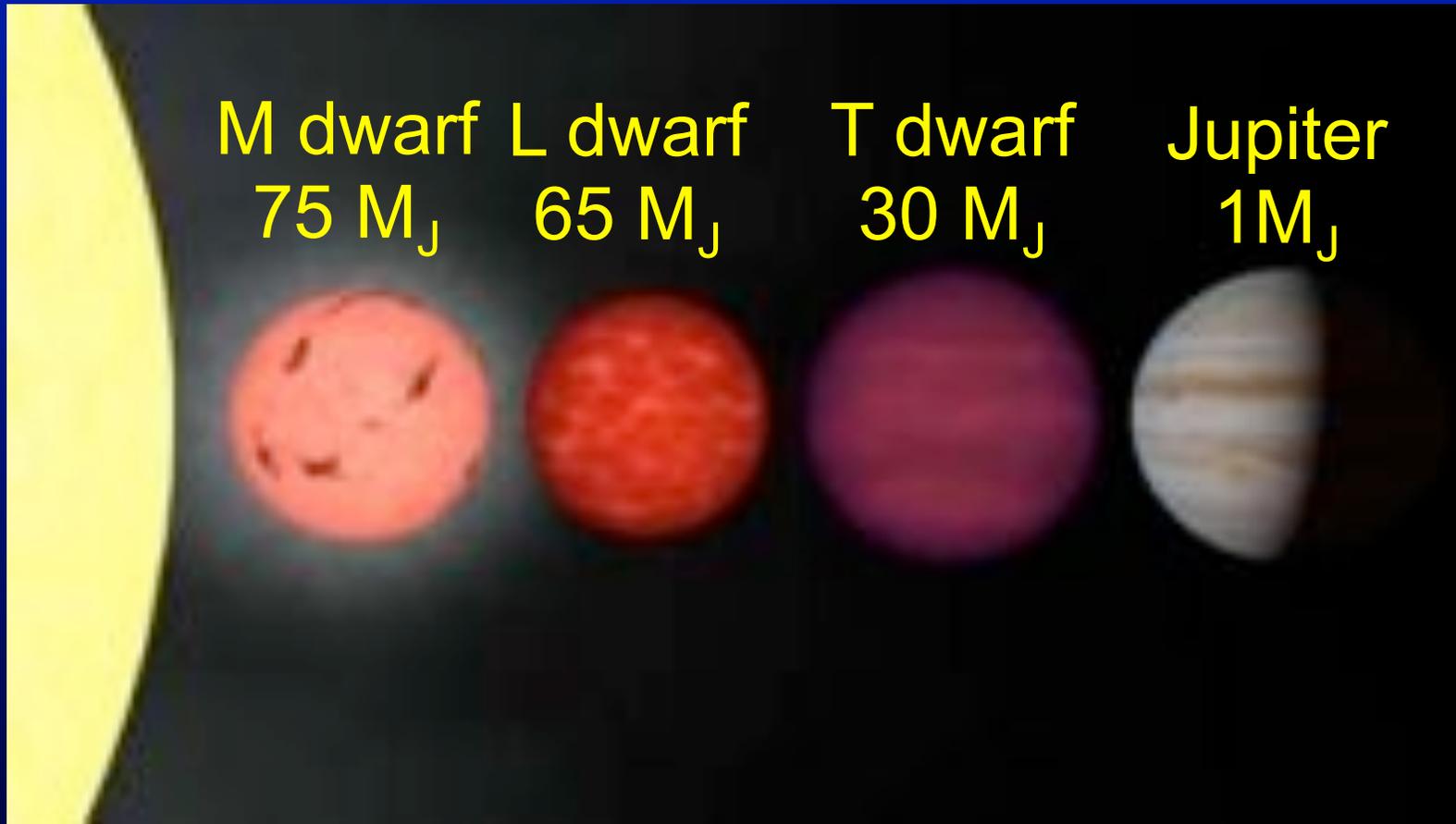
Uncertainties include

- disk temperature
- mass infall rate from surrounding natal envelope
- final planet masses

More easily fragments into brown dwarfs than planets

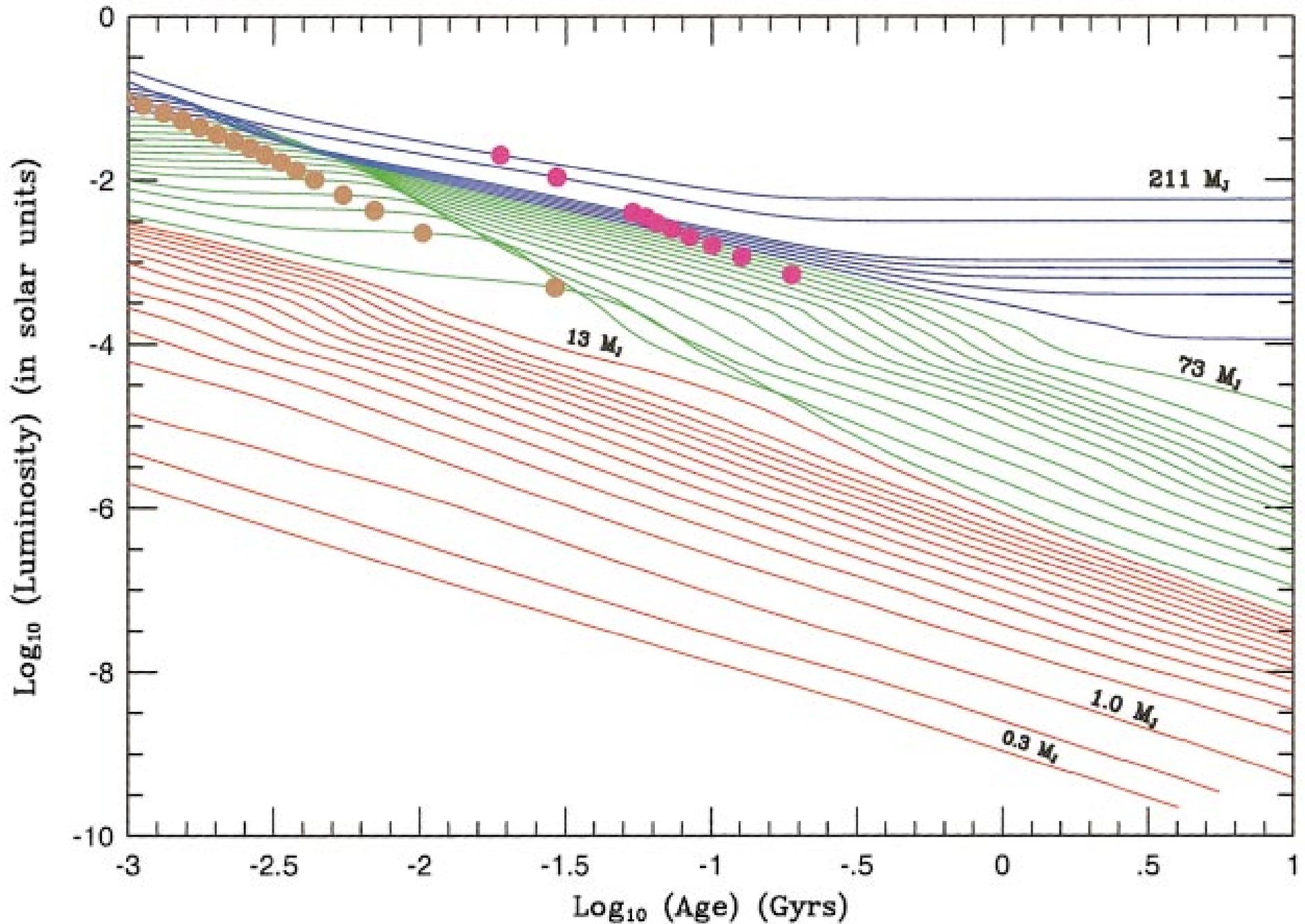
Degeneracy pressure



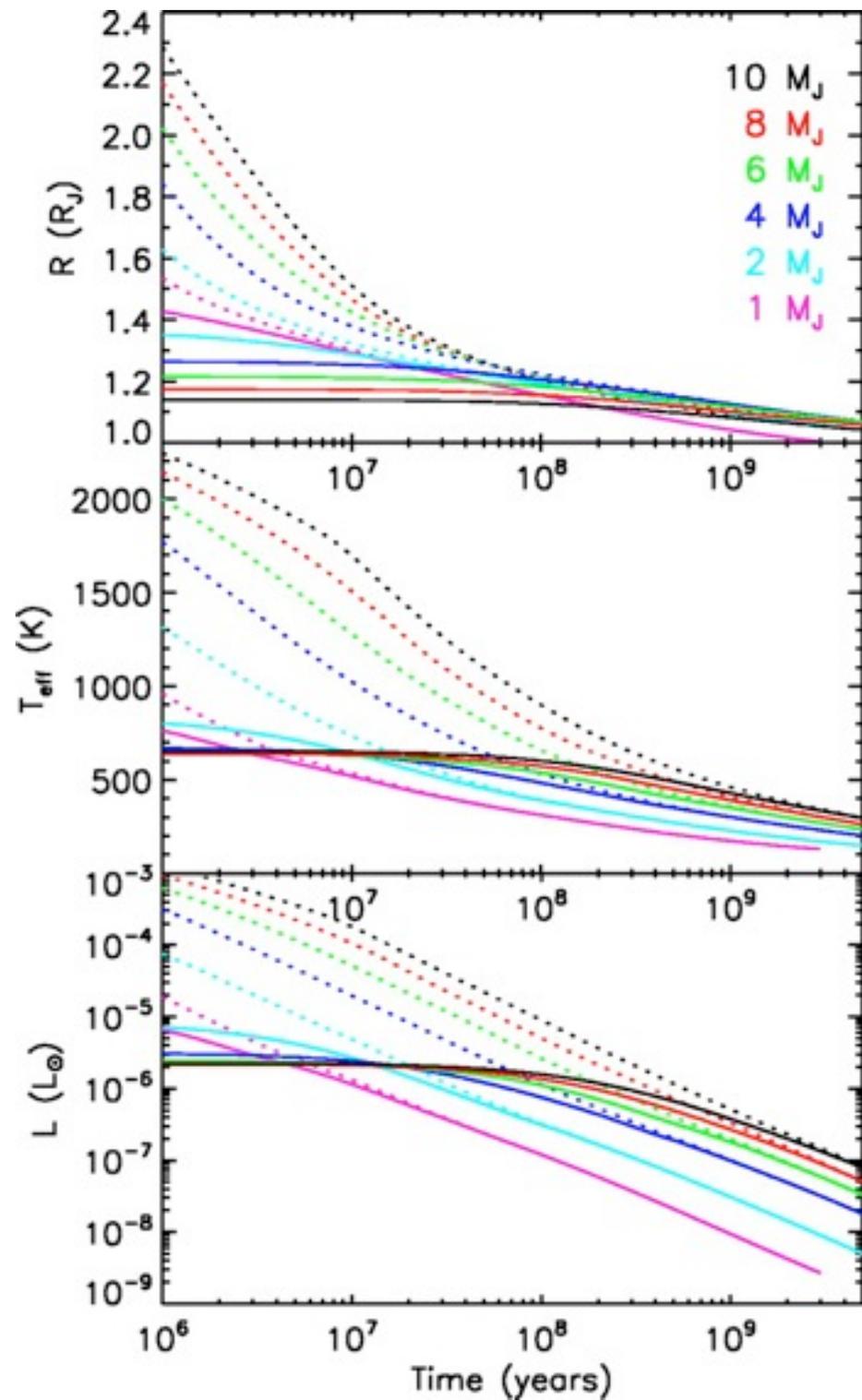
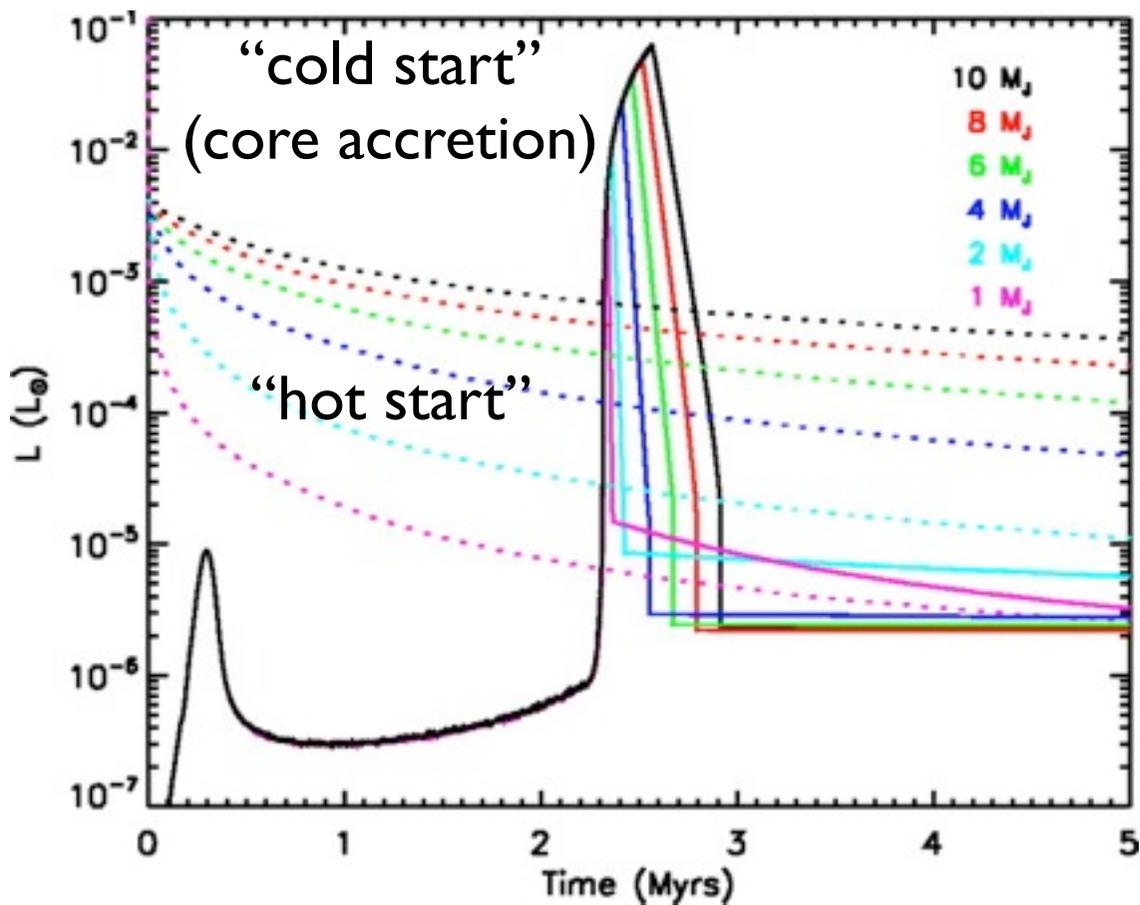


The new spectral classes
O B A F G K M L T

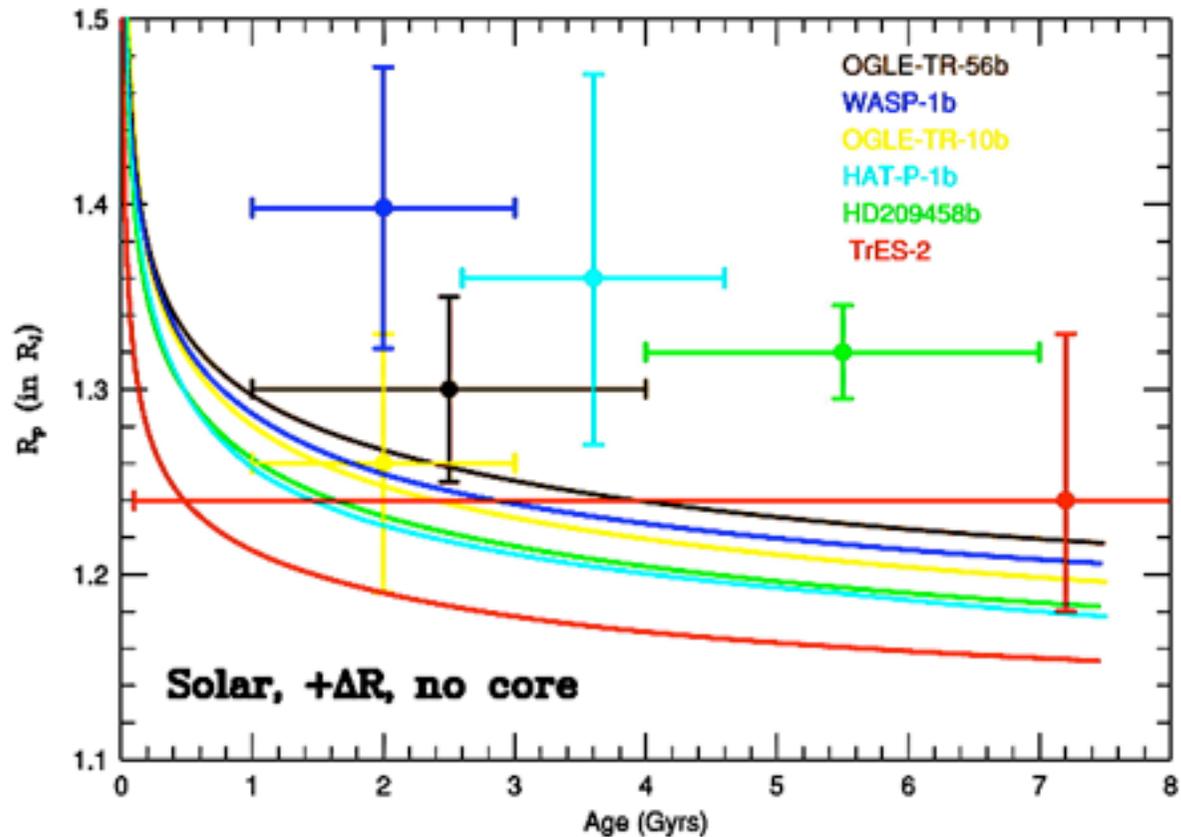
Cooling curves (standard “hot start”)



Early evolution uncertain



Hot Jupiters are inflated

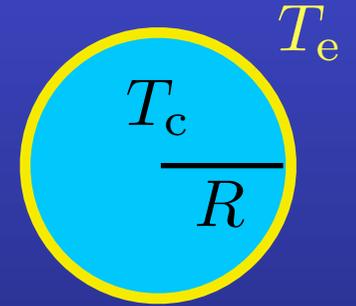


Transit radii $>$ Theoretical radii

How much = How long ago

Radiative cooling: $L = \sigma T_e^4 4\pi R^2 = -Nk \frac{dT_c}{dt}$

Not completely degenerate: $R \sim R_J \left(1 + \frac{kT_c}{\epsilon_F} \right)$



Isentrope: $s_e(T_e, P_e \sim g/\kappa_e) = s_c(T_c, P_c \sim GM^2/R^4)$

3 equations
in 3 unknowns \rightarrow
 T_e, T_c, R

$$L \propto t^{-24/17}$$

$$T_c \propto t^{-7/17}$$

$$R \uparrow T_c \uparrow t \downarrow L \uparrow$$

using more accurate analytic formulae from Burrows & Liebert 93

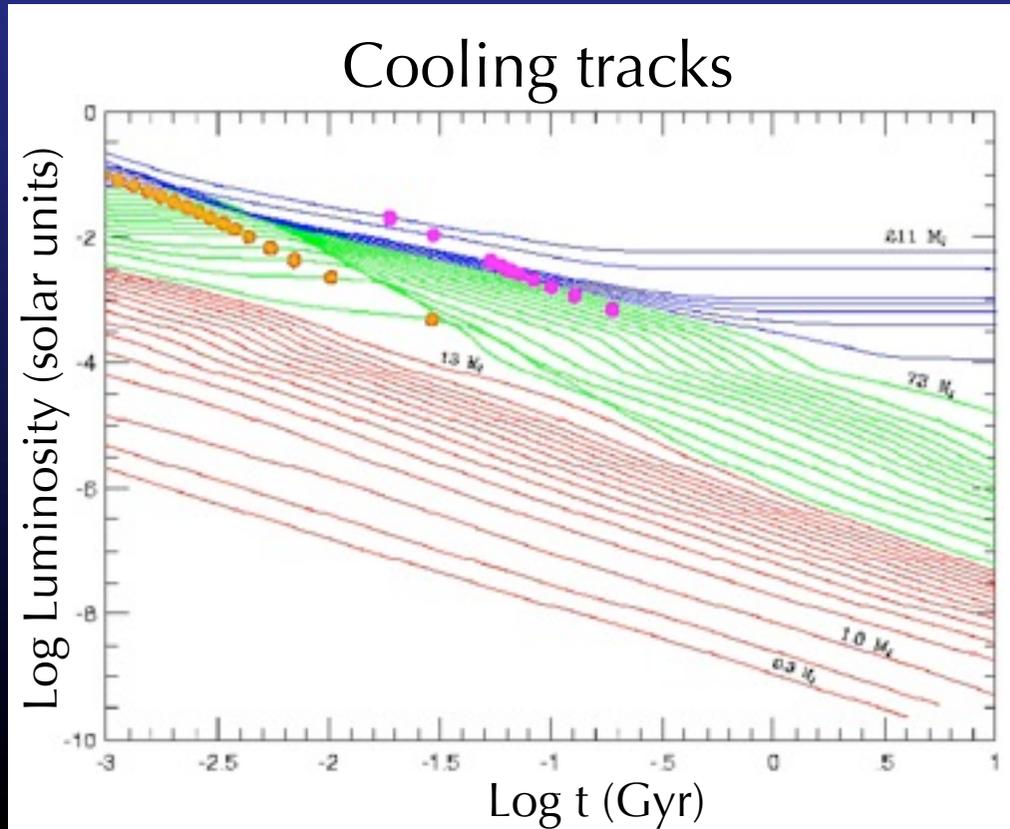
to increase R by 30%,

$$t \sim 2 \times 10^7 \text{ yr}$$

$$L \sim 2 \times 10^{26} \text{ erg/s}$$

vs. numerical $L \sim 6 \times 10^{26} \text{ erg/s}$

Burrows et al. 07



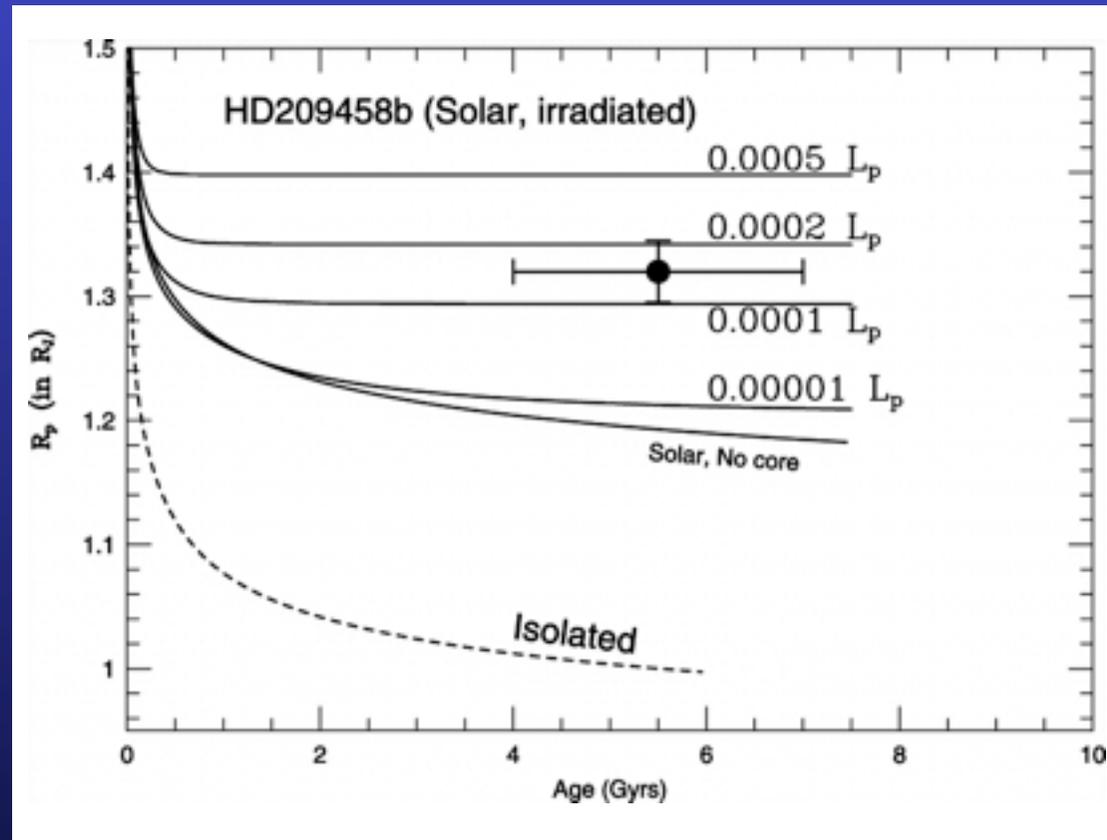
“Easy” problem

Compare required L
 6×10^{26} erg/s

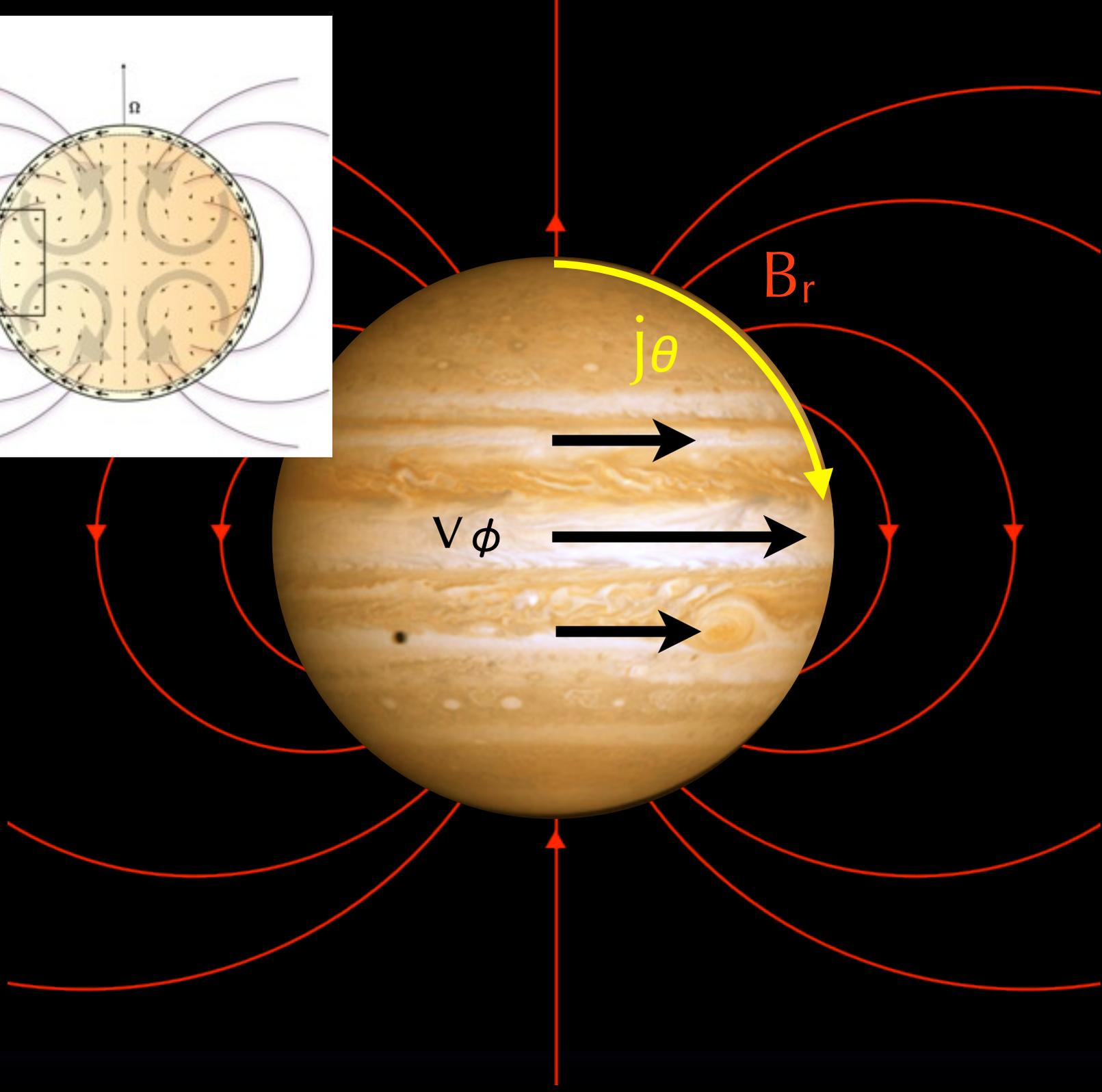
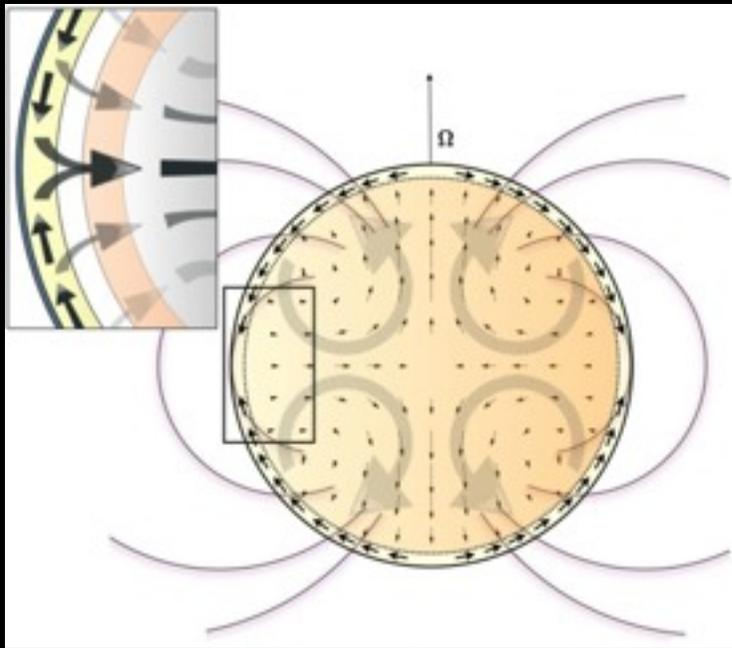
to

Incident L

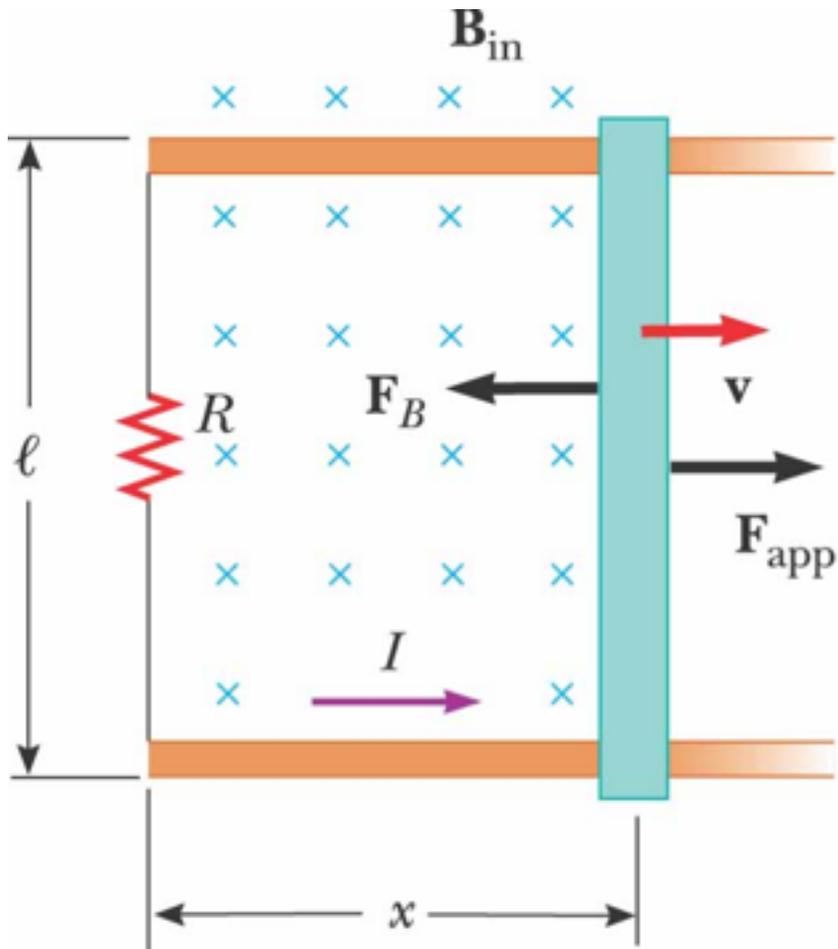
$$\frac{L_*}{4\pi a^2} \pi R_p^2 A \sim 3 \times 10^{29} \text{ erg/s}$$



Even “easier”: When planet is irradiated,
actual required L $\sim 4 \times 10^{25}$ erg/s



Induced Current \Rightarrow Ohmic Power



(a)

©2004 Thomson - Brooks/Cole

$$\mathbf{F} = \frac{q}{c} \mathbf{v} \times \mathbf{B}$$

$$\varepsilon_{\text{emf}} = W/q = F\ell/q$$

$$I = \varepsilon_{\text{emf}}/R = \varepsilon_{\text{emf}} \frac{\sigma A}{\ell}$$

$$\text{Ohmic } P = I\varepsilon_{\text{emf}} = \frac{v^2 B^2 \sigma \ell A}{c^2}$$

copper $6e7$ S/m

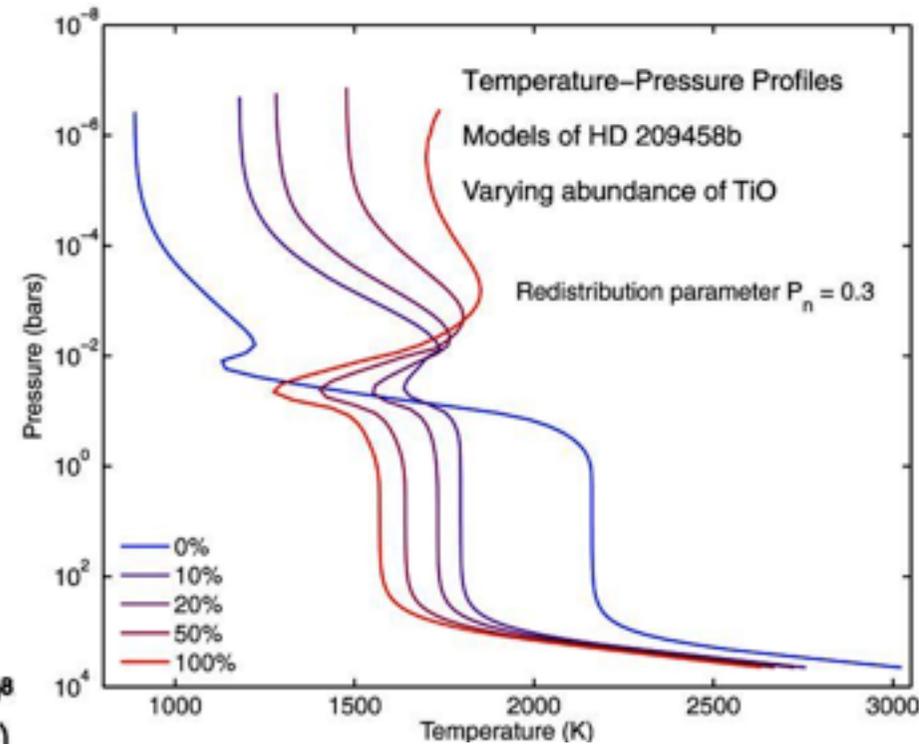
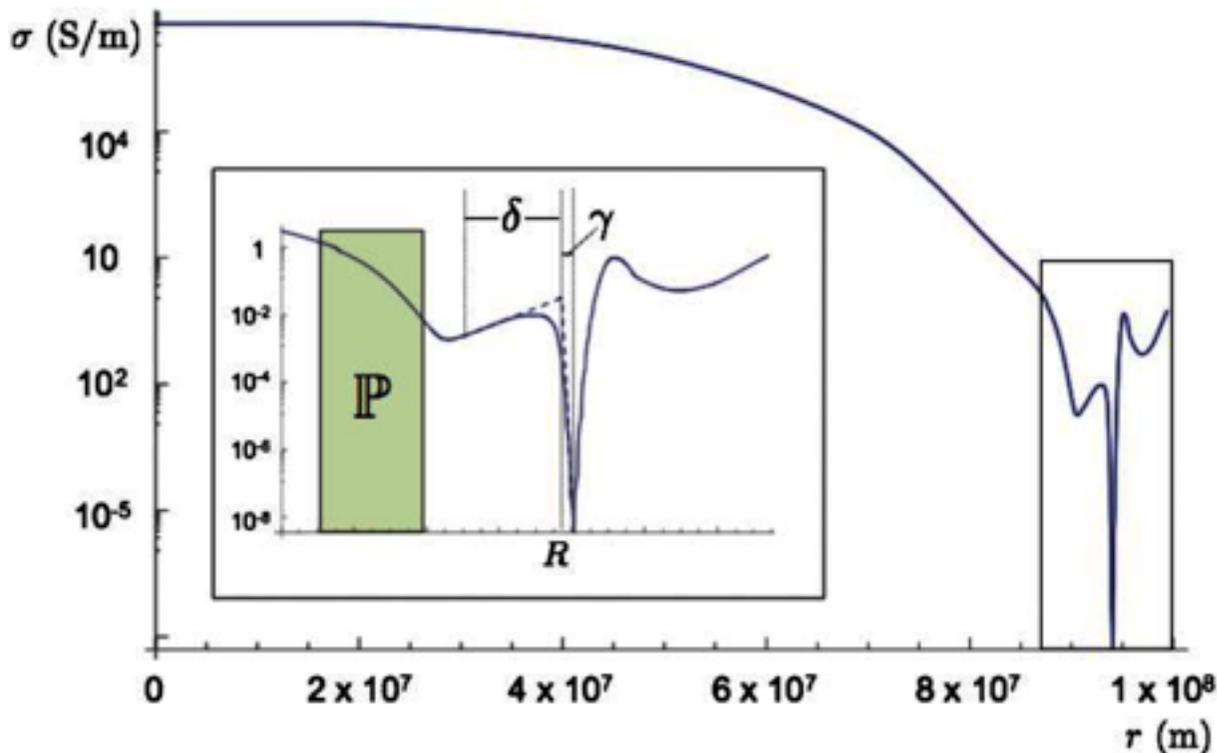
drinking water 0.0005 to 0.05 S/m

$$P = I^2 R$$

$$P = \int \int \int \frac{j^2}{\sigma} dV$$

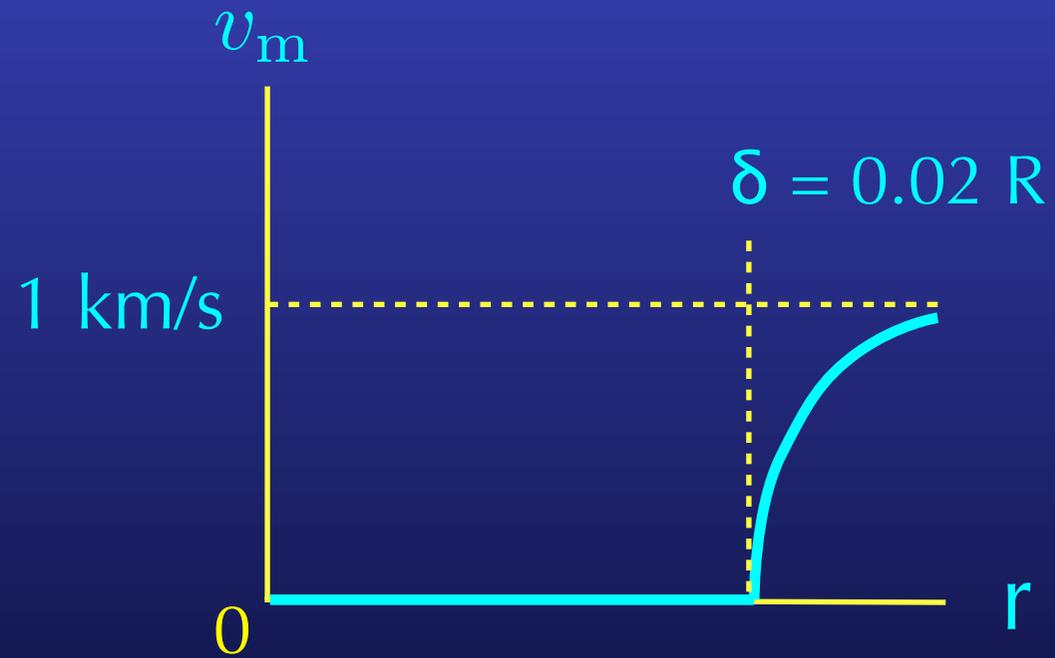
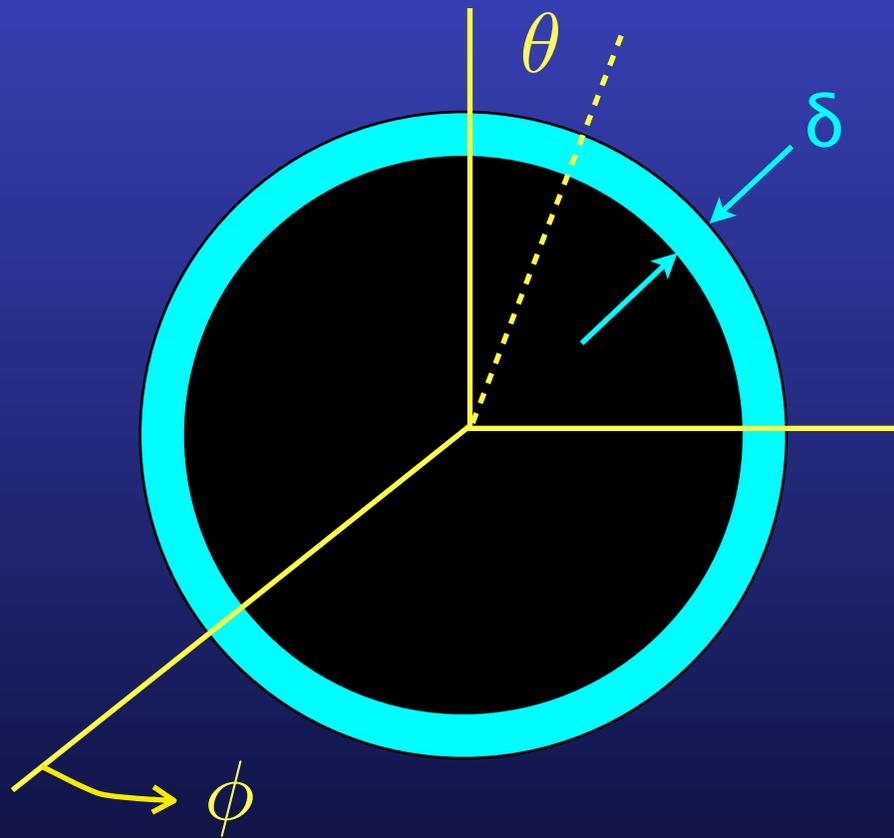
$$\mathbf{j} = \sigma \mathbf{f} = \sigma \left(\frac{\mathbf{v}}{c} \times \mathbf{B} + \mathbf{E} \right)$$

Planetary conductivity



$$\mathbf{j} = \sigma \mathbf{f} = \sigma \left(\frac{\mathbf{v}}{c} \times \mathbf{B} + \mathbf{E} \right)$$

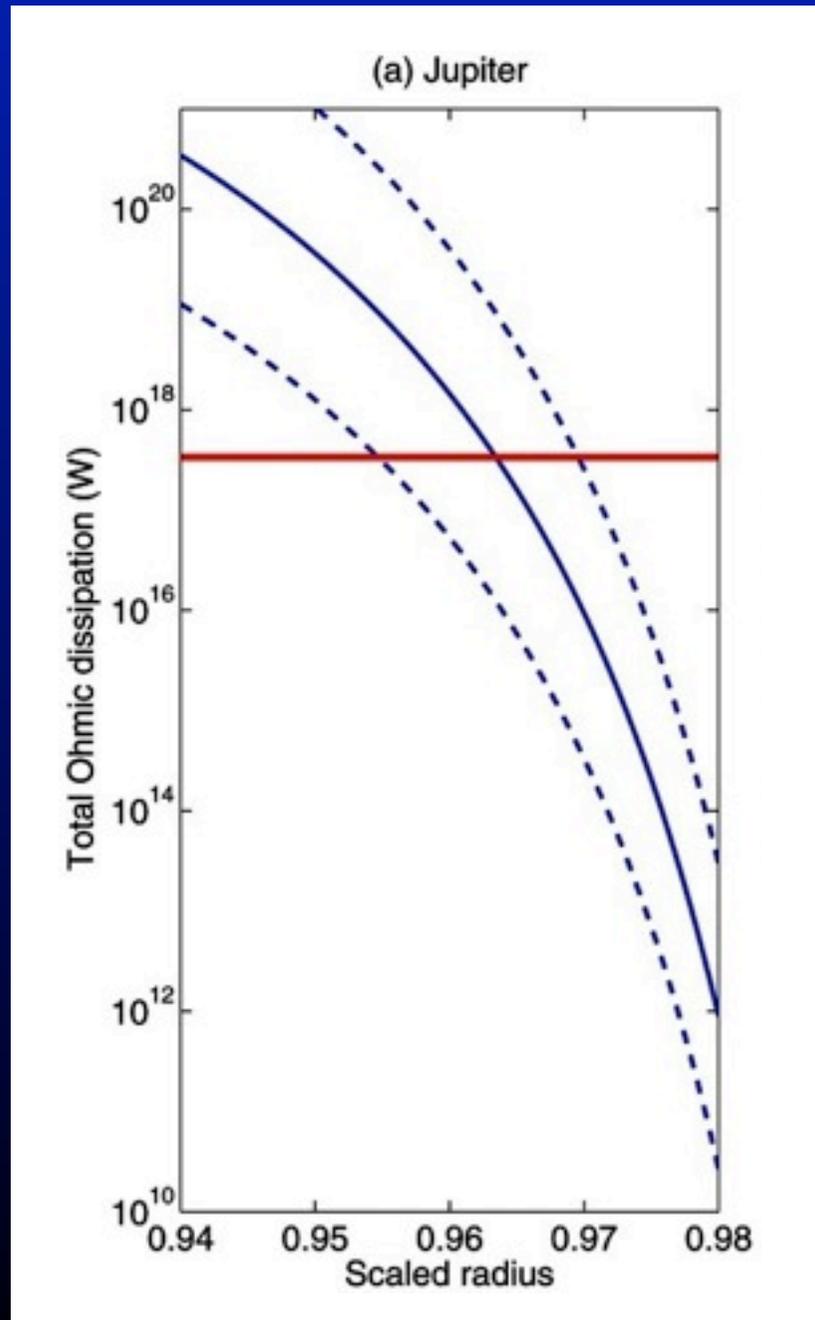
$$\text{delta} \sim 2.3 \times 10^8 \text{ cm} \quad (R \sim 1.05 \times 10^{10})$$



$$\mathbf{v}(r, \theta) = v_m \sin \theta \hat{\phi}$$

$$f \frac{L_*}{4\pi a^2} \pi R^2 \sim \frac{\frac{1}{2} \rho v^2 4\pi R^2 h}{R/v} \Rightarrow v^3 \propto L_*/a^2$$

Differential rotation may only be skin deep

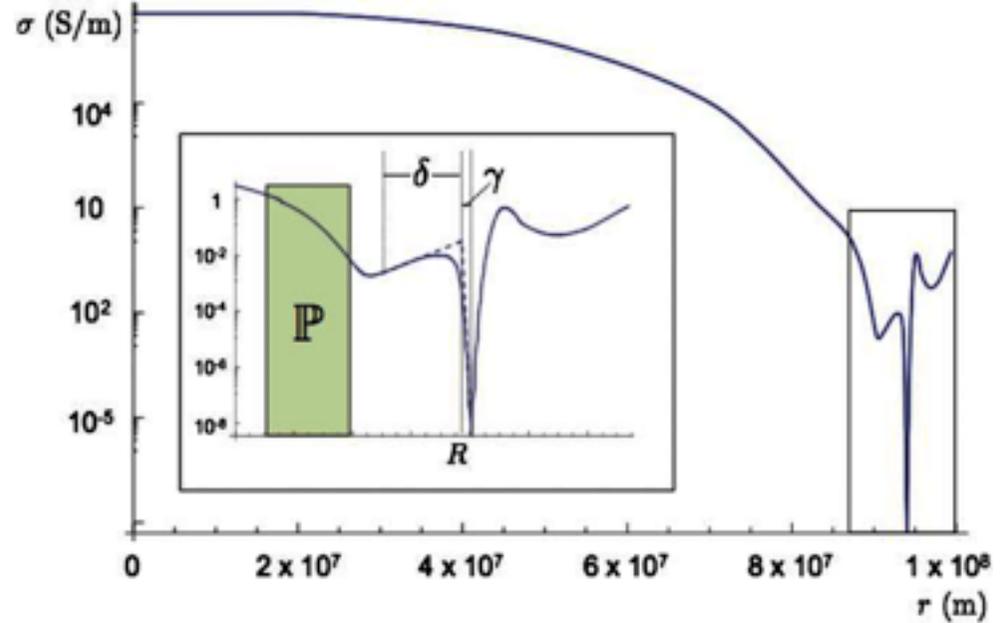
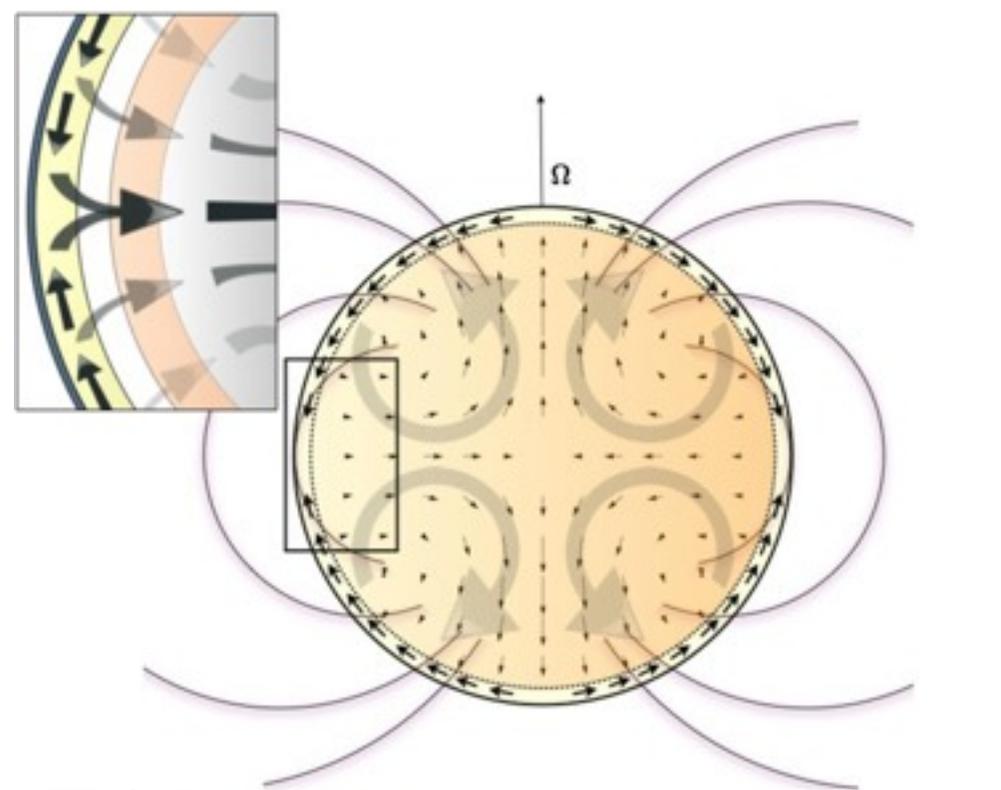


If winds extend too deep,
Ohmic power $>$ internal luminosity

$\delta < 0.03R$ for Jupiter (maybe)

Also Taylor-Proudman theorem,
plus observed stability of B field,
enforces near solid-body rotation in
convective interior (maybe)
[$P(\rho) \Rightarrow v$ constant on cylinders]

Atmospheric Power



$$\mathbf{j} = \sigma \mathbf{f} = \sigma \left(\frac{\mathbf{v}}{c} \times \mathbf{B} + \mathbf{E} \right)$$

$$\sim \sigma \frac{\mathbf{v}}{c} \times \mathbf{B}$$

$$P = \int \int \int \frac{j^2}{\sigma} dV$$

$$\sim \frac{\sigma v^2 B^2}{c^2} 4\pi R^2 \delta$$

$$\sim 8 \times 10^{27} \text{ erg/s}$$

Planet	Y	T_{iso} (K)	Z (\times solar)	$\mathbb{P}[P < 10 \text{ bars}]$ (W)
HD209458b	0.24	1400	1	2.30×10^{19}
HD209458b	0.24	1400	10	7.28×10^{19}
HD209458b	0.24	1700	1	1.14×10^{21}

Power at Radiative-Convective (RC) Boundary

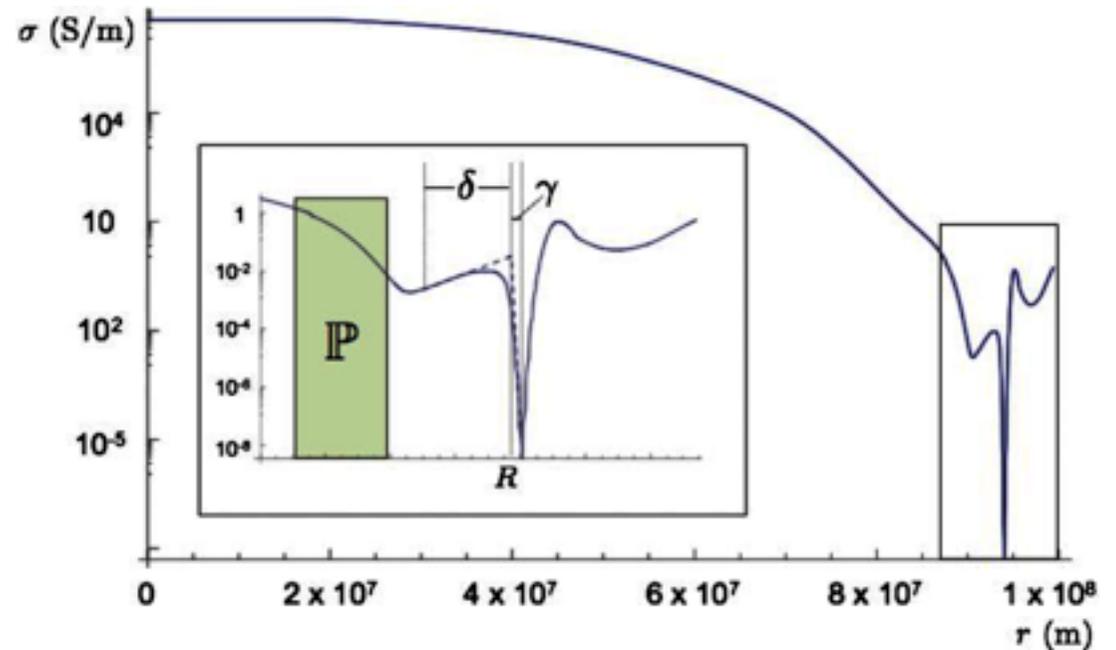


$$P_{RC} = \int \int \int \frac{j^2}{\sigma} dV$$

$$\sim \frac{j^2}{\sigma_{RC}} 2\pi R \times \delta \delta_{RC}$$

$$\sim P \frac{\sigma}{\sigma_{RC}} \frac{\delta_{RC}}{R} \sim 1 \times 10^{25} \text{ erg/s}$$

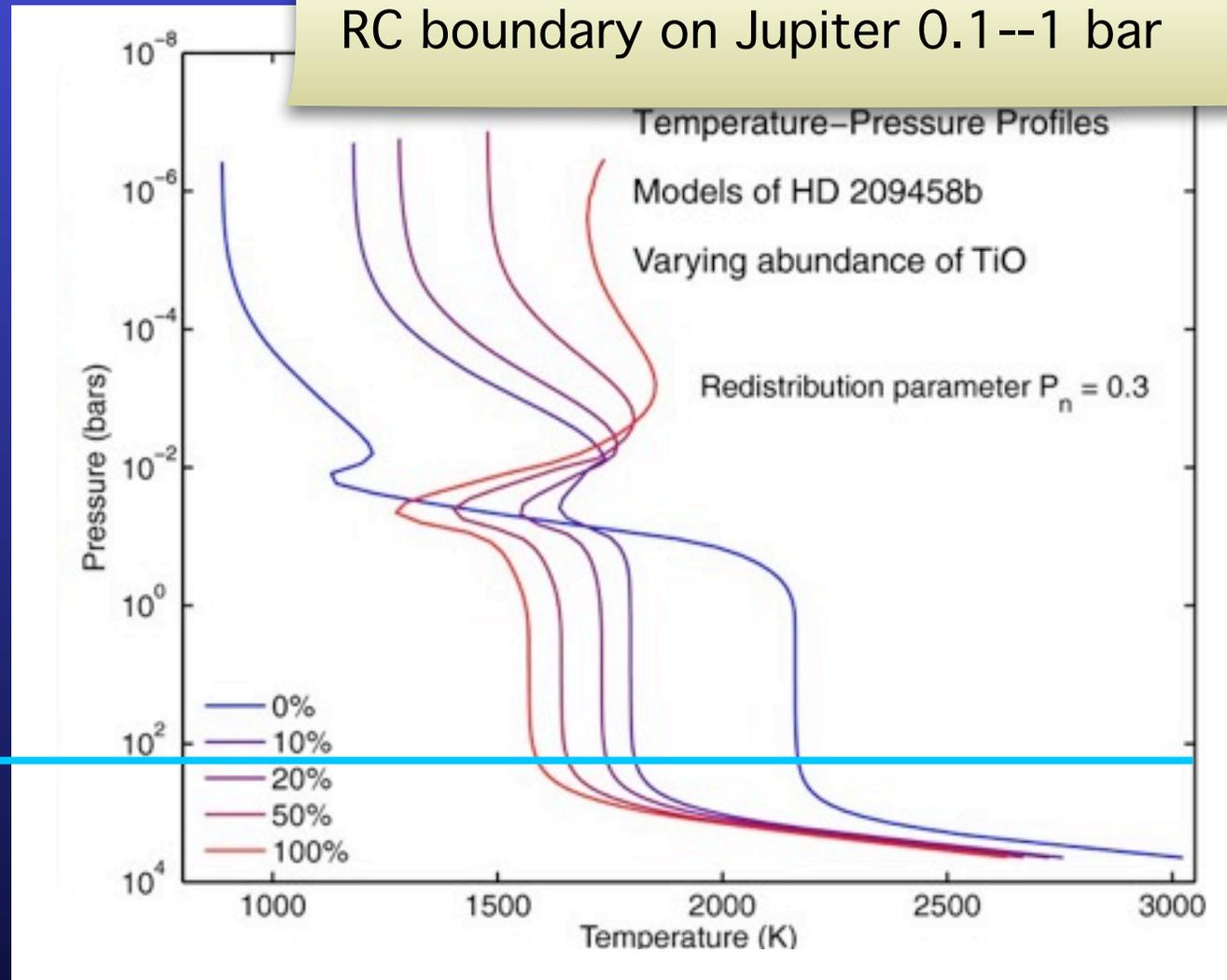
$P [P < 10 \text{ bars}] \text{ (W)}$	$P [P > 10 \text{ bars}] \text{ (W)}$	$P [P > 100 \text{ bars}] \text{ (W)}$
2.30×10^{19}	2.23×10^{17}	1.09×10^{16}
7.28×10^{19}	7.06×10^{17}	3.43×10^{16}
1.14×10^{21}	1.01×10^{19}	5.60×10^{17}



How much extra power and where?

Where :
convective
interior

Radiative-
convective (RC)
boundary ←



$$\text{Specific entropy } s = s_{\text{RC}} \approx s_{\text{core}}$$

$$\Rightarrow R(s, M)$$